

Exploring Elementary School Teachers' Understanding of Number Concepts and Arithmetic Operations

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Abstract. *This study aims to explore elementary school teachers' understanding of the concept of number and arithmetic operations, and its impact on their mathematics teaching practices. In this phenomenological study, six teachers with more than three years of teaching experience in Probolinggo Regency, East Java, were involved as participants. The results showed that there were significant differences in teachers' understanding of mathematical symbols such as '+', '-', '×', and '÷', which often lead to misunderstandings of basic concepts. One of the main findings was the gap between teachers' intuitive interpretations and more scientific mathematical understandings, which we refer to as the Zone of Concept Image Differences (ZCID). Misuse of these symbols has the potential to hinder students' understanding of more complex mathematical concepts. This study highlights the importance of improving teachers' conceptual understanding to support more effective teaching and build a strong mathematical foundation for students at the elementary school level.*

Keywords: arithmetic operations, teacher understanding, number concept, ZCID

1. Introduction

In the world of education, the role of teachers is central in building a strong foundation of mathematical understanding for students, especially at the elementary school level. A good understanding of the concept of numbers and arithmetic operations is an important foundation for mathematical development at the next level (Ginsburg, et al., 2008). These concepts help students build numerical skills and develop logical and analytical thinking skills needed to solve problems (Chin & Pierce, 2019). However, many studies (such as Ball, et al., 2008; Hill & Ball, 2005; Ma, 2020) show that teachers' understanding of number concepts is often influenced by everyday intuition and is not always in line with the scientific mathematical approach. This creates a gap between theory and practice that can ultimately affect the effectiveness of teaching (Der-Ching & Hung-Jin, 2019). Furthermore, misunderstandings in the use of mathematical symbols, such as the operation signs '+', '-', '×', and '÷', often become obstacles in conveying concepts correctly to students (Douglas et al., 2020).

Mathematics education at the elementary school level plays a very important role in building a foundation for understanding more complex concepts later on. At this stage, children learn various basic concepts such as numbers, arithmetic operations, and relationships between numbers that will affect their ability to think logically and analytically (Chin & Pierce, 2019; Douglas et al., 2020). A strong understanding of basic mathematical concepts allows students to handle greater challenges in advanced mathematics education, such as algebra and geometry (Anghileri, 2005). If this foundation is built well, students will be better prepared to face further academic challenges (Ma, 2020). Therefore, focusing on basic mathematics education is crucial to ensure that students can gain solid knowledge and are able to cope with complexities in

the future (NCTM, 2000). Understanding the importance of the role of teachers in this regard is a priority, considering that they are the main facilitators in the learning process (Der-Ching & Hung-Jin, 2019).

Several studies in the last decade have shown that a strong understanding of basic arithmetic operations not only affects students' mathematics achievement but also strengthens critical thinking, problem-solving, and analytical skills (Chin & Pierce, 2019; Douglas et al., 2020; Ma, 2020). For example, a study by Chin and Pierce (2019) revealed that students who have a better mastery of basic arithmetic performed better in other logic-based subjects. Douglas et al. (2020) asserted that the development of analytical skills acquired through a deep understanding of arithmetic is also correlated with improved problem-solving abilities in everyday life. Therefore, developing elementary school teachers' understanding of number concepts and operations is crucial to ensure that students gain a solid foundation needed in their education (Ma, 2020).

According to Der-Ching & Hung-Jin (2019), there are several main components that are the basis for understanding the concept of numbers as a whole. The first is the basic meaning of numbers, which includes an understanding of the intrinsic meaning of each number. Second, number comparison, which highlights the ability to compare values and magnitudes between numbers. Third, number representation and operations, which refer to various ways of representing numbers and their relationship to mathematical operations. The fourth component is the effect of operations on numbers, where understanding of how operations such as addition, subtraction, multiplication, and division affect numbers is very important. Finally, assessing the reasonableness of computational results involves the ability to evaluate and determine whether the results of mathematical operations make sense or not. By understanding these components, teachers can guide students in building strong numerical skills.

This study aims to investigate elementary school teachers' understanding of the concepts of number and arithmetic operations, and how these understandings are reflected in their daily teaching practices. Teachers' conceptual understanding of number and arithmetic operations can impact their ability to teach students effectively, especially in explaining complex concepts in a simple and understandable way (Anghileri, 2005). Previous studies have shown that many teachers tend to use a procedural approach without delving into the underlying conceptual aspects (Douglas et al., 2020), which can lead to misunderstandings in students. Therefore, this study also focuses on how teachers interpret mathematical symbols such as $+$, $-$, $\times$, and $\div$ in various contexts, considering that inappropriate use of symbols can hinder the development of students' understanding (Ma, 2020).

2. Method

This study used a qualitative phenomenological design to explore elementary school teachers' understanding of the concept of numbers and arithmetic operations. This approach aims to investigate teachers' subjective experiences in interpreting and applying these concepts in the classroom. The study was conducted in Probolinggo Regency, East Java, involving six elementary school teachers as participants. The selected teachers had at least three years of teaching experience, which is expected to provide deeper insights into teaching elementary mathematics.

The research instruments included written tests to measure teachers' conceptual understanding of numbers and arithmetic operations, semi-structured interviews to explore teachers' experiences and views in more depth, and documentation in the form of

learning notes and other related sources. Data were collected through a series of structured data collection activities using purposive sampling techniques, where participants were selected based on their relevance and experience in teaching mathematics.

Data analysis was conducted through steps that included coding, grouping findings based on themes, and interpreting the results to identify gaps between practical understanding and ideal mathematical concepts. Through this method, the study aims to provide a comprehensive picture of how teachers interpret the concepts of numbers and arithmetic operations, as well as their implications in teaching practices.

3. Results and Discussions

Our research findings show that elementary school teachers' understanding of number concepts and arithmetic operations varies significantly, as shown in Figure 1. Some teachers demonstrate a fairly good mastery, with the ability to understand and apply basic concepts in simple learning contexts. However, others face significant challenges when trying to apply these concepts in more complex situations, which require the introduction and use of more abstract mathematical symbols and the ability to explain conceptual relationships comprehensively.

Figure 1 displays a collage of handwritten notes and printed text boxes illustrating various interpretations of mathematical symbols (+, -, x, ÷) by teachers. The handwritten notes include examples of addition, subtraction, multiplication, and division using numbers and symbols. The printed text boxes provide definitions for each symbol: '+' for addition, '-' for subtraction, 'x' for multiplication, and '÷' for division. The notes also show some common misconceptions, such as interpreting '3 apples + 4 bananas' as '3a + 4b'.

Handwritten Notes:

- Addition (+):** Menjumlahkan dua angka. Contoh: $2 + 2 = 4$.
 Subtraction (-): Mengurangkan satu angka dari angka lain. Contoh: $5 - 2 = 3$.
 Multiplication (x): Mengalikan dua angka. Contoh: $4 \times 2 = 8$.
 Division (÷): Membagi angka menjadi beberapa bagian yang sama. Contoh: $6 \div 2 = 3$.
- Printed Text Boxes:**
 - Top Right:**
 - "+" is the addition symbol
 - "-" is the subtraction symbol
 - "x" is the multiplication symbol
 - "÷" is the division symbol
 - Bottom Left:**
 - Example: $3 \text{ apples} + 4 \text{ bananas} = 3a + 4b$
 - Example: $3 \text{ apples} \times 4 \text{ bananas} = 12 \text{ apples}$
 - Example: $3 \text{ apples} \div 4 \text{ bananas} = 0.75 \text{ apples}$
 - Example: $3 \text{ apples} \div 4 \text{ bananas} = 0.75 \text{ apples}$

Figure 1. Example of Teacher's interpretation of the symbol “+, −, ×, ÷”

We also found a Zone of Concept Image Differences (ZCID), or a striking gap between teachers' understanding and the scientific concepts they should have mastered. Table 1 presents the gap between teacher interpretation and mathematical meaning of the symbols “+, −, ×, ÷”. This gap has the potential to affect the way material is delivered in class, potentially negatively impacting students' understanding. One aspect that is quite striking is the misunderstanding of mathematical symbols such as “×” and “÷”. Most teachers tend to view these symbols in a simplistic, and sometimes incorrect, way, which can have implications for students' limited understanding of more complex mathematical concepts.

One of the teachers' misconceptions about the “+” symbol is interpreting expressions such as “3 apples + 4 bananas” directly as $3a + 4b$, which shows the idea that

addition operations can be performed regardless of the context. In scientific mathematics, such additions should involve variables and cannot be equated with simple addition. This error shows a tendency towards overly concrete thinking, leading to ZCID, inhibiting the development of more abstract understanding.

Table 1. The gap between teacher interpretation and mathematical meaning of the symbols “+, −, ×, ÷”

Symbol	Teachers' Interpretation	Symbol Meaning
+	Addition sign	Addition Symbol Increase Symbol Positive Symbol Total Symbol
−	Subtraction Symbol	Subtraction Symbol Negative Symbol Difference symbol Reduction Symbol
×	Multiplication Symbol	Multiplication Symbol Times symbol Replication Symbol
÷	Division Symbol Ratio Symbol	Division Symbol Separator Symbol

Another misunderstanding is seen in the symbol '-'. Many teachers have difficulty in understanding the dual role of this symbol, namely as a subtraction sign and as a negative sign. For example, the operation $5 - (-3)$ is often misunderstood, which shows the confusion between subtraction and negative number operations. Consequently, this misunderstanding has an impact on the way teachers teach negative numbers to students, causing misconceptions in learning numbers and operations.

Misunderstandings about the '÷' symbol are common. Many teachers view this symbol as simply dividing two numbers. However, this shallow understanding ignores more complex concepts, such as dividing fractions (e.g., $\frac{1}{2} \div \frac{3}{4}$). Focusing on procedures without a deep conceptual understanding can hinder students from building broader and more adaptable mathematical skills. Therefore, it is necessary to strengthen conceptual understanding among teachers so that they are able to provide a strong foundation for students' mathematical understanding.

This study focuses on how teachers interpret mathematical symbols in real-world contexts, and how these misconceptions can affect their teaching practices. In this regard, we found that although many previous studies (e.g., Swan, 2001) also investigated teachers' misconceptions, they focused more on specific operations, such as addition and subtraction, without involving other more complex mathematical symbols. Meanwhile, our study explores broader understandings, including symbols such as addition, subtraction, and division in various contexts, such as fractions or algebraic variables. While other studies, such as those by Nahdi & Jitsunda (2020) may only touch on aspects of procedural misuse, our study provides a more in-depth analysis of how teachers misinterpret symbols in real-life contexts. We discuss, for example, misconceptions in teaching addition of variables and division of fractions, which not only focus on procedural errors but also broader conceptual errors in their application in the classroom.

This study also introduces the concept of Zone of Concept Image Differences (ZCID) to describe the gap between intuitive understanding brought by teachers and

scientific mathematical understanding. ZCID offers a new perspective in understanding the gap, which is not found in many previous works. For example, in Deringol's (2019) study, although there is a discussion about teachers' misconceptions regarding mathematical symbols, the study did not use ZCID as an analytical framework. This is a significant difference, because ZCID provides a more structured approach in identifying and analyzing gaps in teachers' understanding. Our study differentiates itself by introducing the concept of Zone of Concept Image Differences (ZCID), which is not found in other previous works. ZCID offers a sharper framework to analyze the gap between intuitive understanding brought by teachers and scientific understanding taught in mathematics education. This is a unique contribution of our study that adds a new dimension to the study of teachers' conceptual errors.

Unlike previous studies that often focus on only one or two basic symbols (such as '+' and '-'), our study examines the understanding of a wider range of symbols, such as addition, subtraction and division symbols in more complex contexts (e.g. fractions and variables). This allows us to provide a more complete picture of conceptual errors that occur among teachers. Our findings regarding teachers' misunderstandings of basic mathematical symbols, such as '+', '-', '×', and '÷', are very consistent with previous studies, such as those found by Cavey, et al., (2007) and Turnuklu & Yesildere (2007). Both of these studies identified that misunderstandings of mathematical symbols are common among teachers. In addition, as in Turnuklu & Yesildere's (2007) study, we also found a greater reliance on procedural knowledge than deep conceptual understanding. As reported in Cavey et al., (2007), our study also shows that many teachers tend to rely on everyday experience and intuitive understanding in understanding mathematical symbols. This often leads to applications that are not in accordance with formal mathematical principles. These teachers may have effective understanding in everyday life contexts, but have difficulty in transferring this knowledge into more systematic teaching.

4. Conclusions

Based on this study, it can be concluded that elementary school teachers' understanding of the concepts of numbers and arithmetic operations is greatly influenced by everyday intuition, which often does not align with a more formal understanding of scientific mathematics. This poses a challenge in teaching because there is a gap between teachers' interpretations of these concepts and the meanings that should be understood in the context of scientific mathematics. This gap, known as the Zone of Concept Image Differences (ZCID), contributes to the difficulty in teaching arithmetic operations correctly to students. Therefore, professional development for teachers should focus more on strengthening their conceptual understanding, not just procedural or mechanistic knowledge.

This study makes a significant contribution by exploring teachers' subjective experiences and in-depth understanding of mathematics, especially the concepts of numbers and arithmetic operations. It also highlights how teachers' understanding of these basic concepts influences their teaching methods and impacts the quality of mathematics learning at the primary school level. Additionally, this study provides new insights to improve mathematics teaching practices in primary schools by emphasizing the importance of deeper conceptual understanding.

Based on the research findings, several recommendations can be made to improve the quality of mathematics teaching in elementary schools. First, training that focuses on

mathematical concepts, not just operational procedures, needs to be provided to teachers to deepen their understanding. Second, developing a deeper understanding of mathematical symbols in various contexts will greatly assist teachers in teaching mathematics more accurately and comprehensively. Finally, it is important to ensure that teaching practices carried out by teachers are in accordance with established scientific standards, so that mathematics learning in elementary schools can be more effective and thorough

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