

Meaningful Learning: A Problem-Based Learning Approach to Arithmetic Sequences and Series

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Abstract. *Meaningful learning is crucial as it enables students to develop a deep understanding of the subject matter they are studying. In mathematics education, the focus should shift from rote memorization to problem-solving. Arithmetic sequences and series are mathematical topics that are often challenging for students to comprehend, necessitating a contextual problem-solving approach. This study aims to evaluate the effectiveness of mathematics learning through the implementation of the Problem-Based Learning (PBL) model in fostering meaningful learning experiences for students studying arithmetic sequences and series at state high schools in Depok City, West Java Province. The research employed a case study method involving 32 11th-grade students. Data were collected through observation and written tests. The data analysis included an evaluation of teacher and student activities, as well as students' test results. The findings revealed that both teacher and student activities were categorized as excellent. Moreover, the majority of students achieved the minimum completeness criteria in their test performance. In conclusion, the PBL model effectively facilitated meaningful learning experiences and enhanced students' understanding of arithmetic sequences and series.*

Keywords: *meaningful learning, problem-based learning, arithmetic sequence and series*

1. Introduction

Learning is a process that helps students progress from being unable to being proficient in a specific subject or education in general. Mathematic is one of the subjects that requires a process to be understood properly by students. One of the main characteristics of mathematics is the interconnected nature of its topics. Nendi, Mandur, and Makur (2017) stated that mathematical concepts are interconnected both among themselves and with other scientific disciplines. Furthermore, mathematical concepts are often involved in real-life situations. However, mathematics often poses challenge for students due to various factors, such as the numerous formulas used and the abstract nature of the objects studied (Fauzy & Nurfauziah, 2021). Consequently, mathematics learning presents challenges for both teachers and students.

The impacts of this challenges can be observed in several high schools, including in Keruak City, Nusa Tenggara Barat Province, where student learning outcomes in the topic of sequence and series remain low. Many students are still unable to understand the problem, determine the initial steps for solving it, differentiate between formulas for finding the n th term and its series, and make accurate conclusion (Ariani, Prayitno, Triyaningsih, & Arjudin, 2022). In another high school in Praya City Nusa Tenggara Barat Province, mathematical representation ability of students on the topic of sequences with a visual learning style was 51.59%, which is categorized as very low. Students with an auditory learning style were in the medium category at 64.29%, while those with a kinesthetic learning style were also very low at 49.89% (Ramadhana, Prayitno, Wulandari, & Subarinah, 2022). A similar situation occurred in a state high schools in

Depok City, where data from a particular class showed that 19 out of 32 students scored below the Minimum Completeness Criteria for the topics of arithmetic sequence and series, which was set at 70. These findings indicate that student learning outcomes in this topic are still relatively low, despite its importance as a foundation for other topics, such as annuities in economics. Therefore, innovative teaching methods are required so that teachers can design and implement effective learning processes to achieve learning objective.

Before creating an appropriate learning plan, teachers should analyze common mistakes made by students. Maryani and Chotimah (2021) stated that common mistakes in the topic of sequences include selecting incorrect data due to misreading questions, incorrect working procedures or drawing incorrect conclusions, inappropriate skill hierarchies influenced by students' understanding of sequences, and missing data due to rushing through problem-solving. Based on this error analysis, teachers are expected to instill mathematical concepts correctly, resulting in more meaningful learning outcomes.

The meaningful learning referred in this study involves not merely memorizing but solving mathematical problems. Gazali (2016) explained that meaningful mathematic learning can be applied through the use of contextual problems as a bridge to students' understanding. This concept suggests that students learn better when the learning environment is created as naturally as possible. Additionally, learning becomes more meaningful when students directly engage in and undergo the learning process themselves, rather than just knowing it. Setyowati and Mawardi (2018) similarly suggested that mathematics learning should start with introducing problems related to daily life to make the process contextual and enable students to discover mathematical concepts independently. Arini and Agustika (2021) also support that in contextual learning, students not only understand existing concept but also undergo a process to derive those concepts. Thus, the meaningful learning implies enhancing the quality of education that require deep comprehension rather than mere memorization. However, the implementation of meaningful learning often falls short of expectations, one reason being the choice of methods that do not align well with the material being taught, as observed by the author in mathematics lessons on sequences in a state high school in Depok City. Based on these observations, the author believes that the teaching of arithmetic sequences and series in 11th grade has not yet applied meaningful learning optimally.

The topic of arithmetic sequence is closely related to phenomena in everyday life and requires various problem-solving methods, necessitating strong problem-solving skills (Hardiyanti, 2016). Fardila and Ardipal (2020) stated that the lecture method aims to ensure students acquire knowledge through activities such as listening and taking notes, but it is less suitable for providing meaningful learning experiences on sequences because there is no synergy between this method and the material. Students only gain knowledge without experiencing what they learn firsthand. To address this issue, the author proposes the implementation of Problem-Based Learning (PBL) in the topic of arithmetic sequences and series.

The PBL model provides opportunities for students to learn through presented problems. Zainal (2022) stated that PBL is characterized by student-centered learning, with real-world problems serving as the starting point for learning, integrating various disciplines, and requiring investigation. The teacher acts as a facilitator, while collaboration and communication among students are essential for problem-solving. Given these characteristics, PBL is considered suitable for providing meaningful learning experiences in the topic of arithmetic sequences and series.

Research on meaningful learning combined with the PBL model is still limited, with most studies on meaningful learning being dominated by the Project-Based Learning (PjBL) model. Therefore, the author aims to offer a new approach through this research. This research aims to describe the effectiveness of mathematics learning by applying the PBL model to provide meaningful learning experiences in the topic of arithmetic sequences and series for students in a state high school in Depok, Jawa Barat Province.

2. Method

This research employed a case study approach, focusing on 11st grade student from particular high schools in Depok City. The participants consisted of 32 students, including 12 males and 20 females. The research utilized various data sources, types, collection techniques, and instruments, as summarized in Table 1.

Table 1. Data collection techniques

No	Data Source	Data Type	Data Collection Technique	Instruments Used
1.	Student	Student's ability to understand mathematical concepts	Written test at the end of the study	Written Test Sheet
2.	Teacher and student	Teacher activities and student activities	Observation	Teacher and student activity observation sheet

To ensure the validity of the learning materials used in this study, a senior teacher with adequate expertise acted as the validator. The validation sheets were used to assess the lesson plan, student worksheets, and final test questions. The validator assigned a score to each indicator based on a scale of 1 to 4, where 1 represented insufficient, 2 indicated deficient, 3 was good, and 4 signified excellent. For the lesson plan validation, the key aspects assessed included the opening activities, main activities, and closing activities, all structured according to the PBL model. During the opening activities, the teacher's role was to introduce the learning objectives, assess students' prerequisite knowledge, and engage students by relating the lesson to real-life problems related to arithmetic sequences and series. In the main activities, five indicators were observed: (1) the alignment of the learning steps with the PBL model, (2) group learning facilitated by student worksheets, (3) the teacher's assistance to students facing difficulties, (4) the teacher's guidance during group discussions, and (5) the management of group presentations. The closing activities focused on the teacher's ability to help students draw conclusions from the lesson and effectively close the learning session.

In addition to the Lesson Plan, student worksheet validation was also conducted to support the implementation of the PBL model. The validation sheet for the worksheets contained six indicators: (1) whether the worksheets prompted students to find concepts, (2) the use of language that was clear and easily understood, (3) the guidance provided to help students discover concepts, (4) the alignment of the material with core competencies, (5) the appropriateness of the content for students' cognitive abilities, and (6) encouragement for group collaboration. Furthermore, the validation of the final test questions was crucial to assess whether these questions were suitable for evaluating students' understanding. The validation sheet for the final test included five indicators: (1) the clarity and appropriateness of the language used, (2) the alignment of the test items with the learning objectives, (3) the clear distinction between questions and expected

answers, (4) the suitability of the content for the school level and grade, and (5) the clarity of instructions for students. Once all the instruments were validated, the data from the validation sheets were calculated according to the guidelines.

$$v = \frac{S_v}{S_{max}}$$

With v = Expert validator assessment results
 S_v = The total score obtained from expert validators
 S_{max} = number of indicators

Validity criteria were categorized as shown in Table 2.

Table 2. Validity criteria for research tools and instruments

Interval	Category
$3 \leq X \leq 4$	Highly valid
$2,5 \leq X \leq 3$	Valid
$2 \leq X < 2,5$	Less valid
$1 \leq X < 2$	Invalid

(Adopted from Arikunto with modifications, 2010)

Following the validation of the instruments, the researcher used them to conduct two learning sessions, each lasting 90 minutes. These sessions were observed by a senior teacher to assess both teacher and student activities in the context of the PBL model. The activities were divided into five phases, each corresponding to specific teacher and student actions. The first phase involved the teacher leading students to orient them to the problem and hypothesis, delivering the learning objectives, and encouraging students to engage with the problem. In the second phase, the teacher organized students into small groups of 5 to 6 members. During the third phase, the teacher provided guidance as students conducted group discussions. In the fourth phase, the teacher assisted students in completing their reports and arranged for group representatives to present their findings to the class. Finally, in the fifth phase, the teacher helped students reflect on their problem-solving process, guided them toward the correct solution, reinforced the lesson content, and administered a quiz.

The students also participated in these five phases. In the first phase, students listened to the teacher's explanation, observed phenomena, and developed hypotheses about the given problem. In the second phase, they worked in groups as instructed by the teacher. In the third phase, students conducted experiments and analyzed the results. During the fourth phase, students engaged in group discussions, analyzed the data, presented their findings, and participated in class discussions by responding to the work of other groups. In the fifth phase, students analyzed and discussed the results, improved their problem-solving strategies, engaged with the teacher's reinforcement of the lesson content, and applied their understanding to complete a quiz. The observations of teacher and student activities were recorded using observation sheets, and the data collected were analyzed using predefined calculation guidelines.

$$p = \frac{S_p}{S_{max}} \times 100\%$$

With p = percentage of observer assessment results
 S_p = the total score obtained from the observer
 S_{max} = maximum score

The level of implementation of each phase was calculated as a percentage, with the results categorized as shown in Table 3.

Table 3. Level of implementation of teacher & student activities

Interval	Category
$85 \leq p < 100$	Excellent
$71 \leq p < 85$	Good
$56 \leq p < 70$	Enough
$41 \leq p < 55$	Deficient
$0 \leq p < 40$	Insufficient

(Adopted from Arikunto with modifications, 2009)

At the conclusion of the study, a final test was administered to assess students' understanding of arithmetic sequences and series. The test consisted of 4 word-problems, each connected to real-world problems. Students had 60 minutes to complete the test, which aimed to measure their ability to apply the concepts of arithmetic sequences and series in problem-solving. Student performance was evaluated based on a written test rubric, and each student's score was converted to a range of 0–100. Finally, the classical learning completeness of the students was calculated using a formula to determine the percentage of students who achieved mastery in the topic.

$$KBk = \frac{S_k}{S} \times 100\%$$

With KBk = Percentage of Classical Learning completion

S_k = The number of students who obtained a score ≥ 70

S = Total number of students

3. Results and Discussions

The learning process commenced with an opening activity, followed by the main learning phase where students were organized into groups to discuss worksheets provided by the teacher. These worksheets served as a guide for students to explore the problem. Students were introduced to an arithmetic sequence and series problem involving the Harapan Bangsa Stadium, which has a capacity exceeding 40,000 seats. Specifically, the stadium's seating arrangement includes 92 seats in the first row, 104 seats in the second row, 116 seats in the third row, with each subsequent row increasing by 12 seats. Students were tasked with identifying known values from the problem, determining the number of seats in any given row, and drawing conclusions about the characteristics of arithmetic sequences.

The initial phase, student orientation to the problem, was demonstrated as the teacher presented a conceptual problem related to sequences. Once students grasped the concept of arithmetic sequences, they were required to find the number of seats in specific rows and generalize their findings into a formula for the n th term. Subsequently, students derived the formula for the sum of an arithmetic series and applied it to determine the total number of seats up to a particular row. The organizational phase was evident when the teacher divided students into groups to collaboratively solve the problems outlined in the worksheets.

During the group discussions, the teacher provided guidance, facilitating individual and collective investigations. Whenever a question arose, students were encouraged to seek answers from their peers within or outside their group. The phase of developing and presenting findings was observed as the teacher randomly selected two groups to present their work while others offered feedback. The problem-solving process concluded with

the teacher's clarification of the students' presentations. The learning session ended with a joint conclusion of the day's material and an outline of the next topic.

The observations of teacher activity indicated alignment with the lesson plan. The evaluation of teacher performance yielded an 84.61% score in the first meeting and 88% in the second, resulting in an average of 86.30%, categorized as excellent. Table 4 shows a slight reduction in the teacher's role in guiding individual and group investigations from the first to the second meeting, suggesting that students became more independent as they adapted to the PBL model.

Table 4. Results recapitulation from observing teacher activities

Indicators	Meeting Score	
	I	II
Leading student orientation to the problem and hypotheses	12	13
Organizing student to learn	12	14
Guiding individual and group investigations	19	18
Developing and presenting the work	4	4
Analyzing and evaluating the problem-solving process	8	8
Percentage of average value	84.61	88
Category	Good	Excellent

Observations of student activities reflected an improvement in engagement. The first meeting showed an average score of 81.67%, categorized as good, which increased to 92% in the second meeting, placing it in the excellent category. The overall average score was 86.83%, maintaining the excellent classification. However, as depicted in Table 5, a slight decline was noted in student activity related to developing and presenting work from the first to the second meeting. This was attributed to more time spent on group investigations during the second session, leaving less time for presentations.

Table 5. Recapitulation of observation results of student learning activities

Indicators	Meeting Score	
	I	II
Student orientation to problems	8	8
Organizing students	16	18
Guiding individual and group investigations	12	15
Develop and present work results	5	4
Analyze and evaluate the problem-solving process	8	14
Percentage of average value	81.67	92
Category	Good	Excellent

A final assessment using story problems in arithmetic sequence and series confirmed the findings, as outlined in Table 6.

Table 6. Summary of evaluation results

Information	Score
The highest score	100
Lowest value	26
Number of values	2371
Number of students who took part in the evaluation	32
Average	49
Number of students who completed	23
Classical completeness	72%

The implementation of meaningful learning through the Problem-Based Learning (PBL) model emphasizes active student engagement in constructing and understanding concepts, which can then be applied to real-life problem-solving. To achieve these learning objectives, students need to develop the ability to solve arithmetic sequence and series problems that are relevant to everyday life. This aligns with Gazali (2016), who argued that meaningful mathematics learning is facilitated by using contextual problems as a bridge to enhance students' understanding of mathematical concepts. Gazali's perspective highlights that learning is more effective when the environment is as natural as possible, allowing students to experience and undertake learning actively rather than merely acquiring knowledge passively. Importantly, this requires students not only to understand but to construct knowledge themselves, a process effectively supported by the PBL model.

The PBL model creates an environment where students learn by engaging with presented problems. Lestari and Projosantoso (2016) confirm that PBL introduces contextual problems requiring students to apply analytical skills for problem-solving. This model is rooted in the scientific approach, guiding students through recognizing and formulating problems, investigating solutions, and drawing conclusions, which they then present both verbally and in writing (Mutia, Budi, & Serevina, 2014). Kurnia, Hamdi, and Nurhayati (2015) also emphasize that PBL encourages active student participation and critical thinking, fostering the development of higher-order thinking skills and effective problem-solving. From these perspectives, it can be concluded that PBL serves as an effective approach to provide meaningful learning experiences.

The synergy between the PBL model and the topic of arithmetic sequences and series is particularly evident in 11th grade learning. Djalal (2017) suggested that effective learning outcomes are achieved when teachers apply instructional models suited to the nature of the material. PBL's characteristics employing real-world problems, fostering active learning, using diverse resources to stimulate creativity, and creating a fun and supportive learning environment are conducive to developing students' creative thinking through problem-solving (Handayani & Koeswanti, 2021). Thus, the PBL model's focus on constructing knowledge to address real-life problems aligns well with the nature of arithmetic sequence and series learning.

Arithmetic sequence and series topics naturally lend themselves to real-life applications, making them well-suited for PBL. The methodical approach required to solve these problems complements the PBL model, which engages students in a student-centered learning process. Through PBL, students build cognitive structures by integrating prior knowledge with new concepts, thereby enhancing their ability to solve problems. This makes the PBL model particularly effective for implementing meaningful learning in the context of arithmetic sequences and series for 11th grade students.

The successful application of the PBL model in meaningful learning for arithmetic sequences and series is reflected in the observation data. Table 4 shows the results of teacher activity observations, and Table 5 shows the results of student learning activities, both achieving excellent ratings. Notably, teacher guidance in individual investigations decreased from the first to the second meeting. This change indicates that students became more accustomed to the PBL process and required less guidance over time. Similarly, student activity in developing and presenting results slightly declined in the second meeting as more time was spent on investigation stages, reducing the time available for presentations. Despite this, the evaluation results show a classical completeness rate of 72%.

Analyzing these outcomes reveals the effectiveness of the PBL model in achieving meaningful learning. According to Rohmawati (2015), learning is effective when students demonstrate motivation, preparedness, and active participation, and when teachers deliver high-quality material. Student enthusiasm was observed through active participation, attentive listening, questioning, and engaging in discussions, creating a lively classroom environment. Teacher readiness was evidenced by thorough preparation, including lesson plans (RPP), student worksheets (LKPD), and test questions, validated as suitable for learning by expert validators. These combined observations support the conclusion that the PBL model effectively facilitates meaningful learning in arithmetic sequences and series.

4. Conclusions

The use of the Problem-Based Learning (PBL) model in teaching arithmetic sequences and series has proven to create a meaningful learning experience for students. This approach not only facilitates better understanding of the topic, as evidenced by the majority of students meeting or exceeding the minimum competency criteria, but also enhances student engagement and enthusiasm. Through the PBL model, students demonstrate increased creativity as they navigate and solve complex problems. Additionally, the model fosters collaborative skills as students learn to effectively organize their groups and manage task distribution, promoting teamwork and shared responsibility.

Moreover, the competitive nature of group work within the PBL framework motivates students to strive for excellence, contributing to a dynamic and stimulating learning environment. This competitive spirit, coupled with collaborative efforts, helps solidify their comprehension and allows for a deeper understanding of the material. Ultimately, the implementation of the PBL model on the topic of arithmetic sequences and series transforms the learning experience into one that is not only more engaging but also more meaningful, fostering both academic achievement and the development of essential soft skills.

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