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Solving Geometry Through Triadic Cycles on Harel's Theory: A Systematic Case Study Procedure

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Abstract. *Harel interprets mathematics as the relationship between mental acts, ways of thinking, and ways of understanding that are covered in the triadic cycle. This study aims to examine mental acts, thinking, and understanding in solving geometry problems. Data were obtained from 28 students using written test instruments on geometry materials. This research was analyzed qualitatively using a holistic type case study design with grounded theory data analysis techniques assisted by ATLAS.ti 9 software. Data is collected through tests, observations, documentation, and interviews. Data analysis includes data reduction, presentation, and conclusion drawing. Grounded theory analysis of mathematical procedures using ATLAS ti.9 software consists of the stages of open coding, axial coding, and selective coding found three theme and three categories including the theme of the mental act with the categories of interpreting, explaining, problem-solving. The theme of ways of thinking with multiple interpretations, ways of explaining, and strategic problem-solving. Then found the theme of ways of thinking with the categories of interpretation, explanation, and solution. Grounded theory analysis of systematic procedures produces a hypothetical conclusion. How students think in solving geometry problems affects how to understand new concepts correctly and precisely*.

Keywords: *Atlas.ti 9, case study, geometry, mental act, triadic cycle, ways of thinking, ways of understanding*

1. Introduction

Mathematics learning can be started by critically exploring the phenomena contained in the environment around students. The everyday environment and mathematics are interrelated to develop awareness and critical reasoning (Prahmana & Istiandaru, 2021). This is related to the process of constructing mathematical objects, where the learning situation must allow the occurrence of mental acts, which results in the formation of a continuous flow of thinking, so that flow of thinking is obtained, which leads to an understanding of mathematical objects (Jamilah et al., 2020). In line with the results of research (Nurhasanah et al., 2021), stated that the way students think in solving given problems is influenced by how to understand the concepts that students have learned. The reasoning of human reason always involves mental actions (mental acts) such as interpreting, conjecturing, concluding, proving, explaining, composing, generalizing, applying, predicting, classifying, searching for, and solving problems (Harel, 2008b, 2008a).

Then ways of understanding are a cognitive product when a person performs a mental act (Lim, 2006). The mental act is a characteristic of thinking related to the problem, both externally and internally (Lim, 2006). Through these mental actions will be formed a continuous flow of thinking that leads to one of the targets of the mental object (Amril et al., 2020). The formation of this line of thought is then called the ways of thinking (Çimer & Ursavaş, 2012). When the construction of the flow of thinking occurs and comes into contact with a specific context so that meaning is formed (such as

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concepts, principles, facts), then a flow is formed that leads to understanding (ways of understanding) (Harel, 2008b). Further ways to achieve ways of understanding, ways of thinking, and the beginning of knowledge are depicted in the triadic cycle as presented in Figure 1.

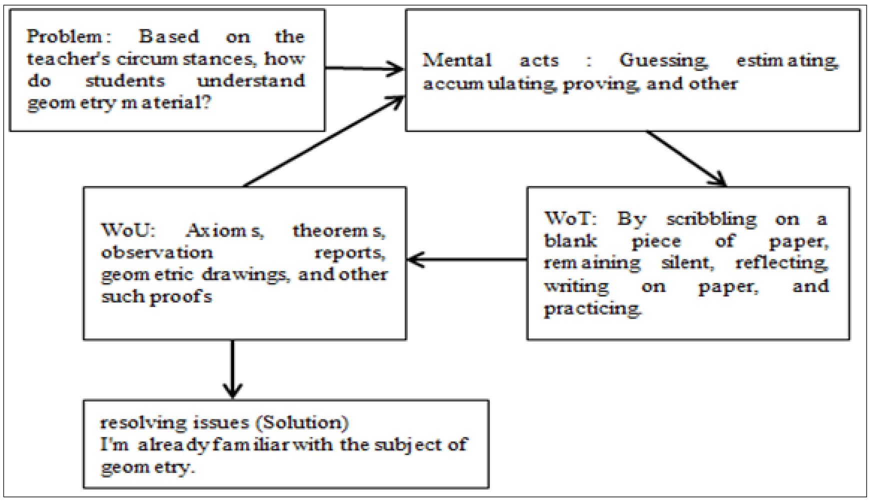

Figure 1. The triadic cycle of learning mathematics

Based on Figure 1, in studying mathematics, students accept problems, perform mental acts, think and then construct knowledge (Guershon Harel, 2001; Jamilah et al., 2020; Koichu & Harel, 2007; Prabowo & Juandi, 2020). A student's learning process departs from the problems built by the teacher. The conditions of the surrounding environment, reading texts, classroom environment, student worksheets, and even teacher stories related to the customs of the local community can be a source of problems built to awaken students' mental acts. About the conditions and situations in the mathematical context, it connects students with mathematics itself. Furthermore, the relationship is called a didactic relationship (Durand-Guerrier & Arsac, 2005; Jamilah et al., 2020; Prabowo & Juandi, 2020; Sulistyowati et al., 2017; Supriadi, 2019). Therefore situations that evoke mental acts in the didactic relationship between students and mathematics are called mathematical didactic situations (Harel, 2008b)

Several research results from previous researchers examined mental acts, ways of thinking, and ways of understanding to fully understand students' thought processes as generalizations of students' cognitive processes, both recursive thinking, explicit thinking, quantitative thinking, and pragmatic thinking, quantitative thinking and visual thinking (Barbosa & Vale, 2015; Becker & Rivera, 2005; Harel, 2020; Lannin et al., 2006; Oflaz & Demircioğlu, 2018; Tallman & Frank, 2020; Zazkis & Liljedahl, 2002).

Several studies related to students' thinking ability in terms of mental acts, ways of thinking, and ways of understanding include research from (Nurhasanah et al., 2021), related to ways of thinking and ways of understanding students in solving problems on vectors reviewed by Harel's theory. Strengthened the results research of (Ikhwanudin et al., 2019) show the mental acts of students found in inclusive classes, namely interpreting, explaining, problem-solving and inferring. Furthermore, the ways of thinking found are multiple interpretations of mathematical symbols, ways of explaining, approaches to problem-solving approaches, and ways to conclude the ways of understanding found, namely the meaning of mathematical symbols, explanations of a problem, solutions, and conclusions.

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Furthermore, Harel's theory regarding the DNR model's duality principle provides a terror-based way of categorizing students' ways of thinking and understanding (Harel, 2008a; Lockwood & Weber, 2015). The principle of duality of the DNR model shows that there is an interdependence in the development of ways of thinking and understanding (Guershon Harel, 2001). In other words, the way of thinking affects the way of understanding and vice versa. Diverse ways of thinking of students can be built by presenting challenges that students must solve in their heads, where various ways of thinking can develop mathematical skills uniquely (Ikhwanudin et al., 2019). Thus, mathematics learning must be presented with situations that give rise to learning needs and repetitive reasoning (Durand-Guerrier & Arsac, 2005; Goos & Kaya, 2020; Stylianides et al., 2004; Stylianides & Ball, 2008). The principle of iterative reason in solving mathematical problems and reasoning must be trained continuously to develop a specific way of thinking and understanding (Çimer & Ursavaş, 2012).

This research focuses on how to solve geometric problems through triadic cycles in Harel's theory. The findings in this study describe mental acts, ways of thinking, and ways of understanding grade IV students at the elementary school level with grounded theory analysis techniques of mathematical procedures using ATLAS.ti 9 software. Based on research that previous researchers have carried out, ways of thinking and ways of understanding are significant to pay attention to in mathematics learning. This is because by paying attention to the mental act, ways of thinking, and ways of understanding, students' potential in learning activities can increase. Researchers also hope that the results of this study can be used as a guide to design teaching and learning activities that can help students to develop optimal ways of thinking and to overcome existing ways of understanding students.

2. Method

This qualitative research is under the characteristics of the scientific environment, researchers as key instruments, qualitative methods, inductive data analysis, developing designs, and assimilation (Creswell et al., 2007). Furthermore, (Charmaz & Belgrave, 2019) recommend combining case studies and grounded theory when the researcher aims to develop a model-theoretic or to obtain hypothetical conclusions based on research data. This study used case studies to investigate a phenomenon (Bassey, 1999; Oaks et al., 2013) regarding students' way of thinking through a limited and deep scope. In this first stage, to capture research data on students, which school students allow to express mental acts, ways of thinking, and ways of understanding students in completing geometry. In this study, the selection of a case study because the researcher wants to conduct an in-depth and detailed exploration of the subject to be studied (Merriam, 1998; Oaks et al., 2013; Roth McDuffie, 2004) in answering research questions that include students' way of thinking to describe students' mental acts, ways of thinking and ways of understanding students. Meanwhile, grounded theory design is used to analyze data in constructing hypothetical conclusions or conjectures (Brigitte Smit, 2002; Creswell et al., 2007; Rambaree, 2013).

Participants

This research was carried out in one elementary school in Sambas Regency, West Kalimantan Province, Indonesia. Furthermore, the participants in this study were 28 students with an age range of grade IV at the elementary school level, ranging from 9 to 10 years. It consists of 15 male students and 13 female students. The results of the

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students' answers are then analyzed to explore the mental act, ways of thinking, and ways of understanding that arise.

Data Collection

The collection techniques used in this study were tests, observations, and semi-structured interviews. Research instruments are used in using data collection techniques. Furthermore, this study uses (a) the main instruments, namely researchers who are research designers, collectors, and data analyzers and interpret the data collected during the research, and (b) supporting instruments consisting of test questions, observation sheets, and interview guidelines.

Written test questions are prepared to collect data to describe students' mental acts, ways of thinking, and ways of understanding. The preparation of instruments in the form of tests in the form of story questions and drawings is carried out with supervision from the supervisor and consideration of the grade IV elementary school mathematics teacher. Before being used to be given to research subjects, written test questions need to be validated to obtain accurate data from the instruments used (Taherdoost, 2018). In this study, for the test instrument, a readability test was carried out first before taking research data in the field. In this case, good data is obtained from research instruments that have been tested properly and correctly (Gelişli & Beisenbayeva, 2017; Hengpiya, 2008; Nordin & Ariffin, 2016; Purnomo, 2017; Thaneerananon et al., 2016).

Observation measures individuals' actions and processes in an observed event (Creswell et al., 2007). Therefore, the observational data can be used to confirm the research data obtained from the tests and interviews. Furthermore, tests and observations, and interviews are carried out to ensure the results of the tests and observations that have been carried out. To record the voices of the students interviewed, researchers used digital voice recorders on WhatsApp videocalls and googled meet(Permana et al., 2021; Septantiningtyas et al., 2021). The interview after the test is necessary to detect the thinking process of the research subject to describe the student's mental act, ways of thinking, and ways of understanding written on the research subject's answer sheet.

Data Analysis

The researcher's data analysis refers to the (Glaser & Strauss, 2017)procedure. The data analysis technique used in this study is the systematic grounded theory with the help of ATLAS.ti 9 software (Rambaree, 2013). Grounded theory systematic procedures are used to obtain a hypothetical conclusion to explore mental acts, ways of thinking, and ways of understanding students at the elementary school level in solving geometry problems (Charmaz & Belgrave, 2019). The analysis techniques in this study used coding and constant comparison. Furthermore, coding is carried out in three stages: open coding, axial coding, and selective coding (Nathan & Walkington, 2017). In this case, the central aspect of the grounded theory according to this system is the writing of memos (Glaser & Strauss, 2017) subsequently by the researcher is taken into consideration from the very beginning of this study. The memo provides a solid basis for the reported research findings (Marzuki et al., 2021).

Validity

In qualitative research, the data is valid if the described data does not differ from the actual conditions(Lewis, 2015). This compares the researcher's data and what occurs to the topic of study to determine whether it is appropriate or relevant. However, the truth

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of the data in qualitative research is plural (not singular) and is also influenced by the researcher's ability to construct the observed or studied phenomena(Creswell et al., 2007). In proving data validation of the study results, the researcher carried out a validation process under the rules of data validation in qualitative research, namely reduction, triangulation, member checking, and contextual completeness (Gall, Gall & Borg, 2007).

3. Results and Discussions

Open coding, axial coding, and selective coding were carried out in the research stage.

3.1 Open Coding

This study uses a grounded theory technique that begins with examining and analyzing mental acts, ways of thinking, and ways of understanding of grade IV students at the elementary school level. Researchers have deciphered the research data by coding each student's answer, then adapted it to the theme and category. Further to organize the data, researchers used ATLAS.ti 9 software. ATLAS.ti 9 has been used by researchers to facilitate the display of data. The open coding stage is carried out by providing codes on each student's answer, data from interview transactions in the google meet application, and video calls via whatapps related to ideas/ideas in solving geometry problems.

Based on the stages of the coding process using ATLAS.ti 9 software on the results of test data analysis, observations, and semi-structured interviews using the google meet application and video calls via WhatsApp for 28 students in grade IV elementary school. In this case, the data of the test results and interviews are extracted and highlighted in the form of sentences which are then presented in a code system using ATLAS.ti 9 software. An example of a code-giving activity on student answers using results ATLAS.ti 9 software is shown in Figure 2.

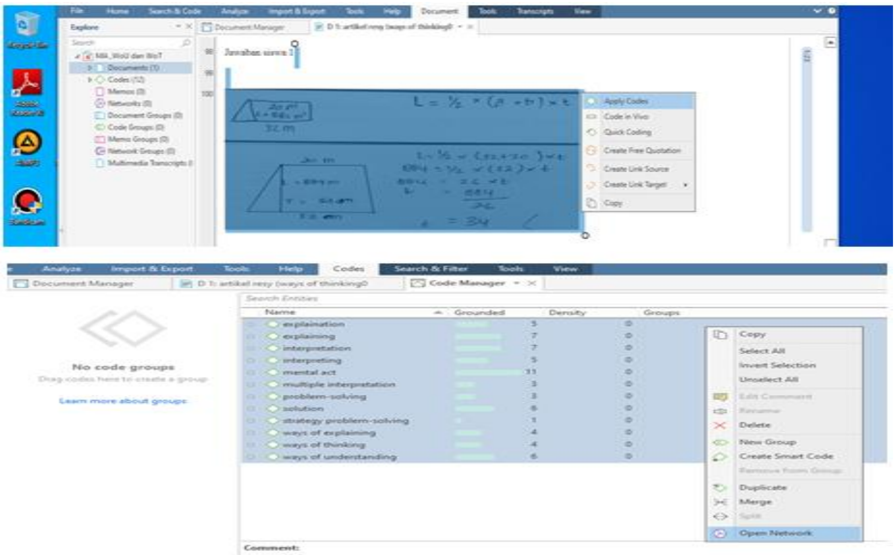

Figure 2. Student answer sheet coding activities with the help of ATLAS.ti 9 software

Based on Figure 2, researchers have carried out open coding stages related to mental acts, ways of thinking, and ways of understanding. In this case, the results of the open coding that has been carried out are three themes and three categories, each characteristic of the student's ways of thinking in solving mathematical problems, especially geometry in the design of the Sambas Malay traditional house presented in Table 1.

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Table 1. Open coding results

Category		Theme
Name	**Grounded**	
Interpreting	86 Code	Mental act
Explaining	89 Code	
Problem-solving	23 Code	
Multiple interpretation	82 Code	Ways of thinking
Ways of explaining	88 Code	
Strategy problem-solving	23 Code	
Interpretation	105 Code	Ways of understanding
Explaination	88 Code	
Solution	55 Code	
Total	639 Code	

Based on Table 1 in the open coding process, three themes, three categories, and 639 codes were found. The themes and categories found in the first theme are mental acts with the categories of interpreting, explaining, and problem-solving. The second theme is ways of thinking with the categories of multiple interpreting, ways of explaining, and strategic problem-solving. The third theme is ways of understanding categories: interpretation, explanation, and solution. The following diagram of the coding process is presented in Figure 3.

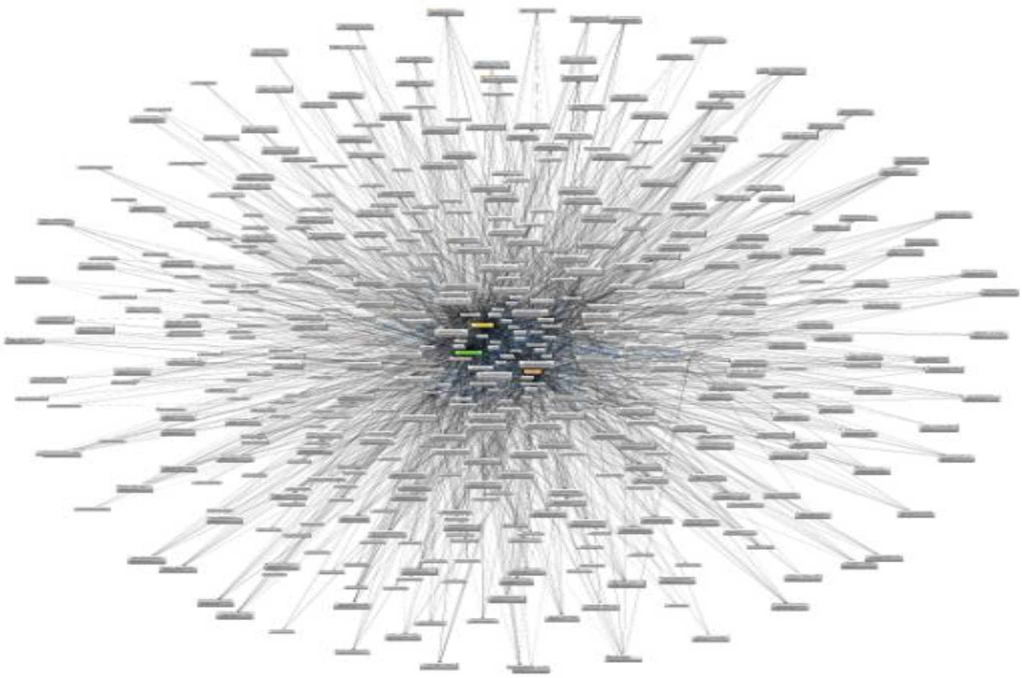

Figure 3. Diagram of open coding process of 28 students

As for the process of open coding mental acts, ways of thinking, and ways of understanding students in solving geometry problems, the results of open coding have been recapitulated in Table 2.

Based on the results of open coding, students who have the skills to reconstruct geometry problems from the knowledge students have can interpret, argue (explain), and develop strategies to solve geometry problems. Then after carrying out the open coding process, it is continued with the next stage of analysis, namely axial coding.

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Table 2. Recapitulation of open coding results

	Themes	
Mental Act	**Ways of Thinking**	**Ways of Understanding**
	Category	
Interpreting	Symbols in mathematics can be interpreted in a variety of ways.	The significance of a symbol
Explaining	What is the best way to explain	The geometry problem is explained in depth.
Problem-solving	Geometric problem-solving approach	The geometry problem is described in detail.

3.2 Axial Coding

The axial coding stage is carried out by choosing a theme or category from the open coding stage. This stage is a central phenomenon causally interconnected with the other categories that make up a logic diagram. The following is presented a diagram of the axial coding process in Figure 4.

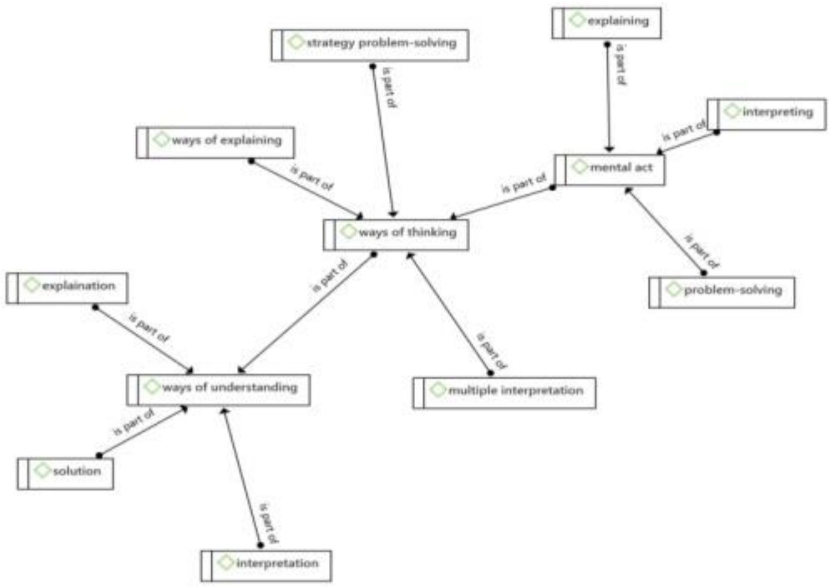

Figure 4. Axial coding process

Based on Figure 4, the axial coding stage of several cluster analyses, then a central phenomenon was identified showing the mental act of students found in class IV elementary school in the categories of interpreting, explaining, and problem–solving. In this case, the mental act is formed from ways of thinking with various categories of interpretation, ways of explaining, and strategic problem solving that is continuously in contact with the context of mathematical problems, namely geometry resulting in the formation of meaning in geometry learning. As a result, a flow is formed that leads to ways of understanding with the categories of interpretation, explanation, and solution. After the axial coding process is completed, the next stage is the selective coding process.

3.3 Selective Coding

In the selective coding phase, a selective coding diagram is built to find the hypothetical conclusion of the data extraction process series. The following is presented a diagram of the selective process of coding mental acts, ways of thinking, and ways of understanding in Figure 5.

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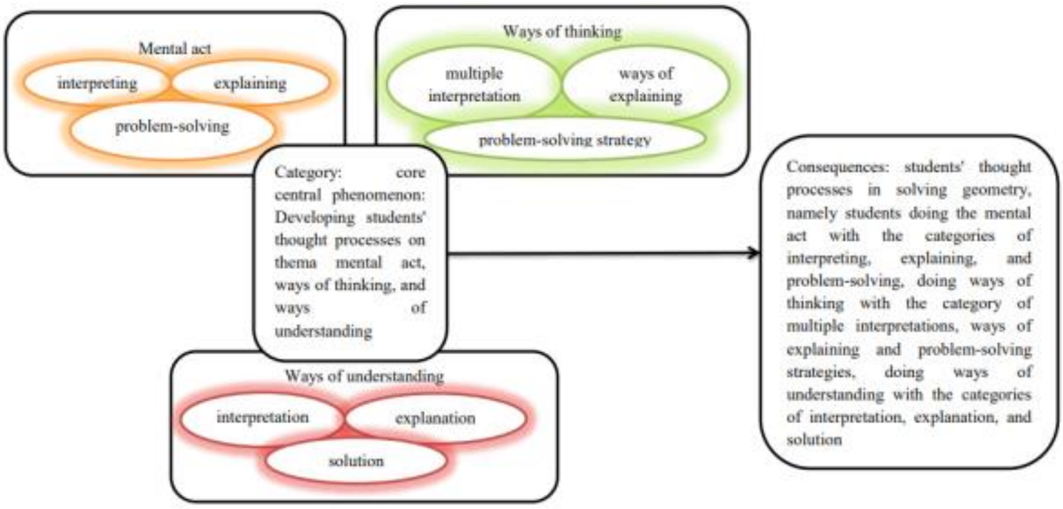

Figure 5. Diagram of the selective coding process

Based on Figure 5, The proper thinking process of students to solve geometric problems with various ways of thinking can build students' mental acts, ways of thinking, and ways of understanding to be more diverse and meaningful. A hypothetical conclusion is obtained that explains the relationship between the categories in the paradigm of selective coding. That is, the way of thinking can influence the ways of understanding a new concept or generating the right solution.

Solving geometry problems through the triadic cycle in Harel's theory found three themes, namely mental act, ways of thinking, and ways of understanding. In the mental act, the ways of thinking and ways of understanding are interrelated with categories (interpreting, explaining, problem-solving, multiple interpretations, ways of explaining, problem-solving strategies, interpretation, explanation, and solution) that build the central phenomenon of solving geometry problems through the triadic cycle. Solving problems is part of the student's thought process in solving geometry problems. Solving problems is part of the student's thought process in solving geometry problems. This is in line with relevant theories and research conducted by (Harel, 2008b; Hazzan & Goldenberg, 1997; Johnson & Moise, 1965; Koichu & Harel, 2007; Lockwood et al., 2016; Sbitneva et al., 2014; Sulistyowati et al., 2017) who stated that problem-solving is an early projection for geometric thought processes. Geometric problem-solving processes involving cognitive processes include mental acts, ways of thinking, and ways of understanding (Çimer & Ursavaş, 2012; Harel, 2008a; Ikhwanudin et al., 2019; Ikhwanudin & Suryadi, 2018; Koichu et al., 2013; Lockwood & Weber, 2015; Nirawati et al., 2022; Nurhasanah et al., 2021).

Furthermore, the findings in this study were grade IV students at the school level who performed mental acts, including interpreting, explaining, and problem-solving. In line with research (Ikhwanudin et al., 2019) state that students in inclusive schools carry out mental acts as follows interpreting, explaining, problem-solving and inferring. The findings in this study are the result of the implementation of Harel's theory that when analyzing students' thought processes, it begins with observing the mental act by looking at the type of cognitive product (ways of understanding related to solving the problem at hand), and looking for a common cognitive trait, namely ways of thinking associated with the mental act. It then classifies mental acts, ways of thinking, and paths of understanding based on typical symptoms observed from student test results.

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In exploring mental acts, ways of thinking, and paths of understanding through stages of systematic grounded theory procedures, which include the steps of open coding, axial coding, and selective coding, obtained the conjecture of students' ways of thinking in solving geometry problems through triadic cycles affects how to understand geometric concepts well and precisely. In line with Harel (2013), who stated that the way of thinking could affect the ways of understanding new concepts/situations/problems. They were strengthened by the results research of by (Nurhasanah et al., 2021), which states that each character of ways of thinking and ways of understanding affects mental actions carried out by students

4. Conclusions

Grounded theory analysis of mathematical procedures using ATLAS ti.9 software, including the stages of open coding, axial coding, and selective coding, found three themes: (1) mental act with the categories of interpreting, explaining, and problem-solving; (2) ways of thinking with the categories of multiple interpretations, ways of explaining, and problem-solving strategies; and (3) ways of understanding with the categories of interpretation, explanation, and solution.

The three themes found in this study, which include mental act, ways of thinking, and ways of understanding, are components of students' thinking processes in solving geometry problems, which have three interrelated categories. Grounded theory analysis of systematic procedures produces a hypothetical conclusion in general, where the way students think in solving geometry problems affects how to understand geometric concepts well and precisely.

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