How Pre-Service Teachers Make Sense of Numbers: A Qualitative Study on Arithmetic Instruction in Primary Schools

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ABSTRACT

This qualitative study explored how pre-service elementary school teachers make sense of number concepts and arithmetic operations as part of their preparation for teaching mathematics. In the context of 21st-century learning, pedagogical innovation requires future educators to develop not only procedural fluency but also deep conceptual understanding. The study involved 65 pre-service teachers enrolled in Primary Teacher Education. Data were collected through written reflections, concept mapping, and semi-structured group interviews, then analyzed using thematic analysis. Data analysis was conducted using the phenomenological thematic analysis method to identify key themes and patterns in pre-service teachers' experiences. The findings of the study revealed significant differences between pre-service teachers' understanding and scientific conceptions, referred to as Zone of Concept Image Differences (ZCID). The conclusion of this study shows that teaching the concept of number and arithmetic operations is often constructed narrowly, thus not supporting the development of broader thinking in students. This study underscores the importance of embedding conceptual discourse in pre-service training to foster innovative teaching practices.

Keywords: *arithmetic instruction, concept image, pedagogical innovation, phenomenological approach, qualitative study*

1. INTRODUCTION

Teachers' ability to understand basic mathematical concepts, especially numbers and arithmetic operations, is an important foundation in learning mathematics in elementary schools. A strong conceptual understanding of a teacher will directly affect how they teach mathematics meaningfully to students (Ball et al., 2008). However, various studies show that many pre-service teachers still have limited and tend to be procedural understanding of these concepts (Kim & Albert, 2015; Thanheiser, 2009; Vula & Kingji-Kastrati, 2018). The main problem in this study is the limited conceptual understanding of prospective teachers regarding the concept of numbers and arithmetic operations. Prospective teachers often only understand mathematical operations algorithmically without being able to explain the meaning behind the procedures they perform (Tirosh, 2000). For example, when asked to explain why "$5 \times \frac{1}{2} = 2\frac{1}{2}$", many of them experience confusion in explaining the concept of multiplication with fractions because their previous understanding is more rooted in the habit of memorizing tables and formal procedures (Thanheiser, 2009).

To overcome this problem, pedagogical innovation in teacher education programs is needed. One of the strategies used in this study is a reflective and qualitative approach that allows prospective teachers to reflect on how they interpret and understand the concept of numbers. This strategy was chosen because it has been proven to be able to reveal deep aspects of teacher understanding that are often not visible through quantitative approaches (Creswell & Poth, 2018). This approach is also in line with the needs of 21st-century education, which requires teachers to not only be able to teach but also have deep conceptual understanding and reflective and critical thinking skills (Darling-Hammond et al., 2017).

Previous studies showed that many prospective teachers have misconceptions about understanding number representation and are less able to relate concepts to real contexts (Ardiansari, 2024; Nahdi & Jitsunda, 2020; Thanheiser, 2009). Meanwhile, a study by Zazkis and Leikin (2010) highlighted the importance of involving prospective teachers in exploratory and reflective activities to build a more meaningful understanding of basic mathematical concepts. However, there is still little research that specifically explores how prospective

teachers interpret the concept of numbers and arithmetic operations in the context of elementary education in Indonesia by involving a large group of participants.

This gap is an important foundation for this study, which aims to fill the gap in the literature on how prospective teachers interpret basic mathematical concepts, and how this understanding shapes their pedagogical perspectives in the classroom. Therefore, this study seeks to explore the understanding of prospective elementary school teacher students towards the concepts of numbers and arithmetic operations, and how these understandings are formed during their education process.

To achieve these objectives, this study is guided by three main questions: (1) *How do prospective elementary school teacher students understand the concepts of numbers and arithmetic operations such as addition, subtraction, multiplication, and division?* (2) *How do their learning experiences in lectures and microteaching shape their understanding of these concepts?* (3) *What factors influence the construction of their understanding of the symbols and meanings behind arithmetic operations?*

The results of this study are expected to contribute to the development of teacher education curricula, especially in strengthening conceptual mathematical understanding as part of pedagogical innovation in the 21st-century learning era.

2.   METHODS

This study used a qualitative phenomenological approach to explore in depth the subjective experiences of students in understanding basic mathematical concepts. This approach was relevant to uncovering how conceptual understanding was formed in the context of 21st-century teacher education, especially in building pedagogical innovations based on reflection and active learning experiences (Creswell & Poth, 2018; Moustakas, 1994).

The participants in this study were 65 students of the Elementary School Teacher Education (PGSD) study program in semesters 4 to 6 from a private university in East Java. The sampling technique used was purposive sampling, namely selecting participants who had taken the course "Basic Concepts of Elementary Mathematics" and had participated in microteaching. Of the total participants, 12 students were selected to take part in in-depth interviews based on the results of written answers that showed diversity in understanding the concept. The instruments used consisted of two forms: (1) a written test in the form of an open questionnaire and reflective questions on the concept of numbers and arithmetic operations, and (2) a semi-structured interview guide. The written test was designed to reveal students' conceptual understanding of the symbols +, −, ×, and ÷ in the context of representation, application, and relations between concepts. Five main aspects were assessed in this test, namely: a) Conceptual Meaning: the extent to which students understand the logical principles behind each operation (Kilpatrick et al., 2001); b) Symbolic Flexibility: the ability to apply symbols in various unconventional contexts (Pape & Tchoshanov, 2001); c) Visual and Contextual Representation: how students illustrate operations through pictures or mathematical models (Ainsworth, 2006); d) Relationships Between Concepts: such as the relationship between multiplication and repeated addition (Goldin & Kaput, 1996); e) Teaching Application: how this understanding emerges in teaching simulations or microteaching. Meanwhile, the interview guide was developed to explore the narratives of students' learning experiences, perceptions of teaching materials, and their reflections on applying mathematical concepts during teaching practice.

Figure 1. Research Instruments on Understanding of Number Concepts and Arithmetic Operations

Data collection was carried out in two stages. The first stage was the distribution of written tests to all 65 participants. This test was conducted online with a processing time of 60 minutes. The results of the written test were then analyzed to identify answer patterns and misconceptions. Based on these results, 12 students with the most varied answer profiles were selected for further interviews. The second stage was a semi-structured interview conducted both in person and online. All interviews were recorded and transcribed for data analysis purposes.

Data analysis was conducted through thematic analysis using an open, axial, and selective coding approach (Braun & Clarke, 2006). The researcher identified key themes related to conceptual understanding, learning strategies, and symbolic meaning construction based on participants' learning experiences. To maintain the credibility of the study, method triangulation was conducted between written test data and interviews. Data validity was strengthened by member checking, which was asking for confirmation from participants regarding the researcher's interpretation. An audit trail was also carried out by systematically documenting the entire analysis process. In addition, dependability was maintained through discussion and verification of results by two independent peer researchers.

3.   RESULTS & DISCUSSION

3.1   The "+" symbol

The findings reveal diverse conceptualizations among prospective teachers regarding the "+" symbol. A majority (52) adhered to conventional views, interpreting "+" as representing addition—consistent with early arithmetic learning literature (Behr et al., 1983). Within this group, five described it as an "addition sign," three focused on its function in combining numbers, and one defined it as "combining objects or quantities."

However, closer analysis uncovers notable conceptual variations indicative of potential misconceptions. Two participants referred to the symbol as a "summation sign," focusing on increasing quantity rather than binary operations. One participant described it as a "total sign," highlighting results without clarifying the process. Most notably, one participant misunderstood "+" as a symbol that changes negative numbers to positive, conflating its role with unary operations in integer arithmetic.

These variations reflect a Zone of Concept Image Differences (ZCID)—a gap between the participants' informal concept images and the formal mathematical definition of the "+" symbol (Tall & Vinner, 1981). While some interpretations align with standard definitions, others reveal surface-level or procedural understandings

detached from formal arithmetic structures. This conceptual gap may challenge instructional clarity, especially when explaining more complex arithmetic or addressing student misconceptions.

In response to the item asking for the correct interpretation of "3 + 4 × 2," several students misapplied the symbol by selecting options like "3 added together equals 2" or "3 plus 4 times 2," without applying the correct order of operations. This highlights a misunderstanding where "+" is viewed as an immediate summation tool, disregarding precedence rules. The presence of ZCID here indicates that prospective teachers rely on informal heuristics, lacking alignment with formal mathematical syntax and semantics.

Further, when asked about the distinction between the addition sign and the summation sign, ten participants who were interviewed claimed both symbols were the same, indicating limited awareness of their different mathematical roles (Mulligan & Mitchelmore, 2009). Only two participants demonstrated a nuanced understanding of their distinct functions and contexts—aligning with findings from Ball, et al. (2008), which show variation in teachers' content knowledge.
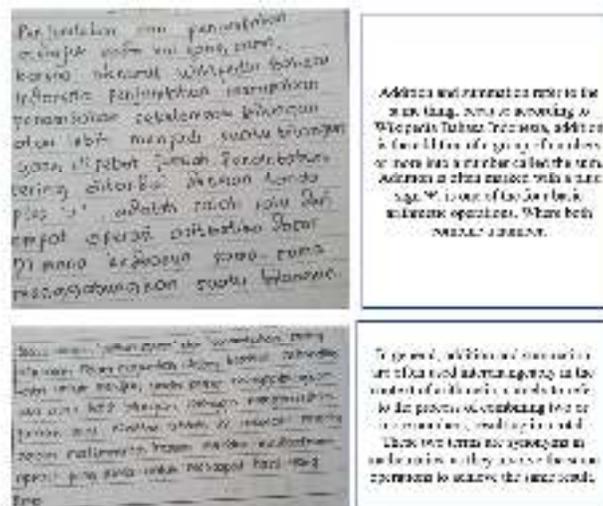
Figure 2. Examples of responses regarding the differences between addition and summation.

These findings underscore not only gaps in symbol interpretation but also highlight previously underexplored nuances in teachers' conceptual frameworks. Thus, this study contributes new insights into the variability of symbol understanding and reinforces the need for deeper conceptual development in mathematics teacher preparation.

3.2 The "−" Symbol

Many prospective elementary school teachers have difficulty solving problems such as $10 − (−4)$ and $5 − (−3)$, where they actually give answers that are smaller than the initial value. This error indicates a misconception in understanding the meaning of the symbol '−'. In the context of understanding symbols in various situations, it was revealed that all participants were not yet able to distinguish between the term's 'minus' and 'negative' in the use of the symbol '−'. They read $10−4$, $8+(−3)$, and $5−(−3)$ as "ten minus four", "eight plus minus three", and "five minus three", without realizing that the term 'minus' refers to the subtraction operation, while 'negative' refers to a number value below zero (Bittinger et al., 2017; Larson & Edwards, 2015; Van de Walle et al., 2018).
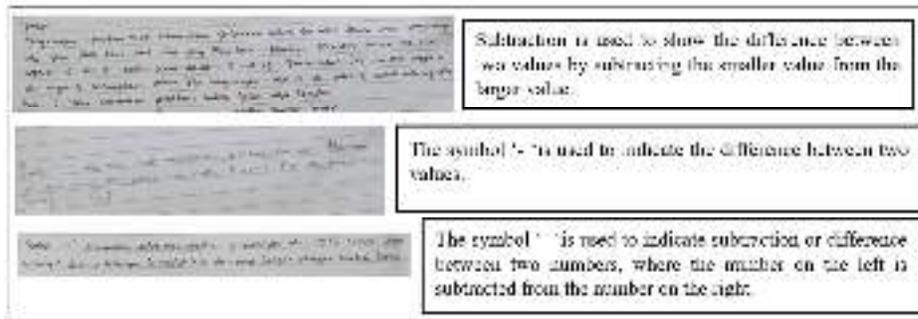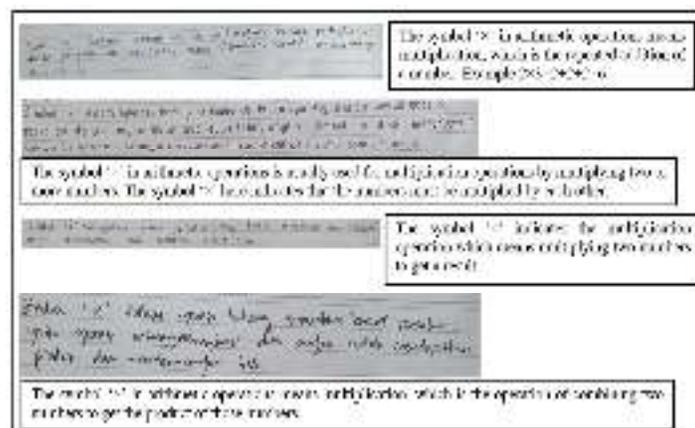
Figure 3. Various meanings of the symbol "−"

If these misconceptions are not corrected and brought into the classroom, elementary school students are at risk of experiencing confusion in distinguishing subtraction and negative numbers, making calculation errors, and having difficulty understanding advanced mathematical concepts. This can also hinder problem-solving skills, reduce interest in learning mathematics, and cause conceptual errors that are repeated across generations.

This gap reflects the Zone of Concept Image Difference (ZCID) phenomenon, where the concept of subtraction image that participants have is limited to the meaning of "taking something". When faced with negative numbers, they are confused about whether to subtract or add. In fact, conceptually, $10 - (-4)$ should be interpreted as $10 + 4$ (Bofferding, 2014). Therefore, a visual or contextual approach is needed to build a more appropriate mental representation that is in accordance with the formal concept.

3.3   The "×" Symbol

When asked to represent 3×6 in various forms (array, equal group, repeated addition, number line), most students were only able to mention repeated addition as the only form of representation. This shows that their concept image of the symbol '×' is still limited to operational meaning alone.

Figure 4. Various meanings of the symbol '×'

Students' understanding of the symbol '×' is generally limited to the interpretation as repeated addition, such as 3×4 explained as $3 + 3 + 3 + 3$ or $4 + 4 + 4$. Even in cases such as -2×3, they still rely on the repeated addition model, for example $-2 + (-2) + (-2)$. However, when asked to explain whether "3 added as many as negative two times" is meaningfully acceptable, they start to get confused. This indicates their limited understanding of the concept of multiplication by negative numbers, which cannot be fully explained by the repeated addition model. Negative multiplication should be associated with the concept of inverses and directions on the number line, not simply repeated addition (Clark & Kamii, 1996; Bofferding, 2014).

This limitation reflects the existence of the Zone of Concept Image Difference (ZCID), which is the gap between students' understanding and the conceptual understanding that elementary school mathematics teachers

should have (Tall & Vinner, 1981). The unfamiliarity with other forms of representation such as arrays (e.g. 3 rows 6 columns) or number lines indicates that previous learning tends to emphasize algorithmic procedures, rather than conceptual meanings visually or contextually. In fact, teachers' understanding of the various meanings of multiplication symbols is very important because it will affect how they teach it in class (Ball et al., 2008; Ma, 1999).

3.4 The "÷" Symbol

Teachers' understanding of the '÷' symbol is very important because it is the basis for teaching arithmetic. However, many prospective teachers interpret it narrowly, only as a command to divide two numbers directly. In fact, conceptually, this symbol can mean division as an equivalent division (partitive), measurement, or ratio. Ignorance of these variations in meaning has the potential to cause difficulties in explaining more complex division concepts, such as fractional division (Ball et al., 2008; Clark & Kamii, 1996).
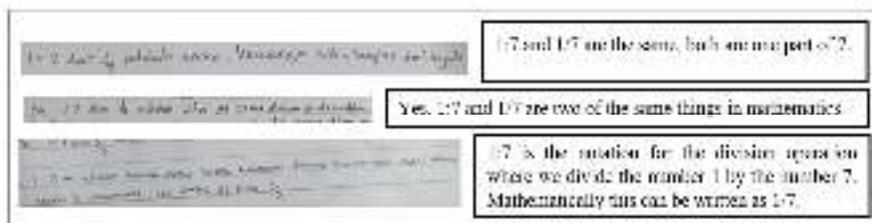
Figure 5. Example of reasons respondents agree that $1 \div 7$ and $\frac{1}{7}$ are two of the same things.

The interview results also showed that 9 out of 10 students said that $1 \div 7$ and $\frac{1}{7}$ are the same thing. Although the final result is the same numerically, the two have different conceptual meanings. The form $\frac{1}{7}$ represents one part of seven equal parts, while $1 \div 7$ is an arithmetic operation that describes the division process. The inability to distinguish symbolic representation from mathematical operations indicates a conceptual gap that has the potential to affect the teaching of fractions in the classroom.

In the context of dividing fractions such as $\frac{1}{2} \div \frac{3}{4}$, mathematically it is interpreted as ½ times the inverse of $\frac{3}{4}$, which is $\frac{1}{2} \times \frac{4}{3}$, (Ma, 1999; Van de Walle et al., 2018). This procedure is often memorized by students as the rule of "multiply by the reciprocal", without understanding why this procedure applies. This indicates the existence of Zone of Concept Image Differences (ZCID), which is the distance between the conceptual understanding that should be possessed and the conceptual image that students actually possess (Tall & Vinner, 1981). This misconception can have an impact on the way they explain and teach fraction operations to students. In the question "Is it true or false? $\frac{1}{2} \div \frac{3}{4} = \frac{1}{2} \div \frac{4}{3}$,", many students answered "true" because they thought that dividing fractions could be directly exchanged with multiplication, without understanding the conceptual meaning behind the operation. This error shows a misconception about the symbol "÷", which should represent the process of dividing the first number by the second number, rather than simply replacing the sign with the usual multiplication operation.

4. CONCLUSION

This study explored how pre-service teachers make sense of arithmetic symbols and operations within the context of primary school mathematics. The findings revealed that many pre-service teachers tend to rely heavily on procedural knowledge rather than conceptual understanding, particularly in interpreting symbols such as '+', '–', '×', and '÷'. Misconceptions were especially evident in the use of division and multiplication symbols with fractions, where surface-level familiarity with algorithms (e.g., "invert and multiply") overshadowed deeper conceptual reasoning. The study also highlighted a significant *Zone of Concept Image Differences* (ZCID) between the formal mathematical meaning of symbols and the mental representations held by pre-service

teachers. These gaps may lead to inappropriate instructional practices that can hinder students' understanding of number operations, particularly when transitioning from whole numbers to more complex number sets such as fractions and decimals.

The implication of these findings suggests the need for teacher education programs to place greater emphasis on conceptual understanding in mathematics. Instruction should provide opportunities for pre-service teachers to reflect on and reconstruct their mathematical conceptions through discussion, model-based reasoning, and varied representations of number operations. Embedding tasks that promote metacognitive awareness about symbolic meanings and their use in context could be particularly beneficial.

This study was not without limitations. First, the sample size was relatively small and drawn from a single-teacher education program, which may limit the generalizability of the findings. Second, while the study focused on symbolic understanding, it did not deeply explore the role of instructional practices or classroom observations. Future research could expand by involving diverse teacher preparation contexts and incorporating longitudinal studies to examine the development of conceptual understanding over time. Despite these limitations, this study still contributes to a growing body of literature emphasizing the importance of conceptual clarity in early mathematics instruction and highlights the critical role of teacher preparation in shaping how future educators approach arithmetic in meaningful and effective ways.

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