



Unraveling Misconceptions: A Study of First-Year University Students' Understanding of Basic Mathematical Concepts

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ABSTRACT

This study investigated the misconceptions held by first-year university students regarding basic mathematical concepts, with a focus on integer operations, fractions, and elementary algebra. Conducted at a private university in Aceh, Indonesia, the research involved 26 students enrolled in the Elementary Algebra course. A diagnostic test consisting of five open-ended questions was administered, followed by structured interviews with 6 students who exhibited notable errors. The findings revealed persistent misconceptions, including misapplication of integer rules, incorrect fraction operations (e.g., adding numerators and denominators directly), and flawed algebraic reasoning. These issues indicate a reliance on procedural knowledge rather than conceptual understanding, often rooted in prior learning experiences. The results highlight the importance of early diagnostic assessments and concept-focused instruction in bridging the gap between school and university mathematics. Addressing these misconceptions is essential to support students' mathematical development and academic success in higher education.

Keywords: *misconceptions, basic mathematics, first-year students, fractions, integer operations, elementary algebra*

1. INTRODUCTION

A solid grasp of fundamental mathematical concepts is essential for first-year university students, as these concepts form the foundation for advanced studies in various disciplines. However, numerous studies have highlighted persistent misconceptions among students regarding basic mathematical operations, particularly with integers and fractions. For instance, Lee and Boyadzhiev (2020) investigated college students enrolled in remedial mathematics courses and found common misconceptions related to fractions. Students often lacked understanding of the basic definition of fractions, struggled with least common denominators and multiples, and misapplied the order of operations. Even when students could recall procedures, they frequently made computational errors due to these underlying misconceptions. Similarly, Permata et al. (2019) conducted a qualitative study to describe students' misconceptions on algebraic prerequisite concepts, focusing on operations with integers and fractions. The research revealed that students often misapplied rules for integer operations and had difficulty understanding the properties of fractions, leading to consistent errors in problem-solving.

Further research by Jarrah et al. (2022) examined seventh-grade students' understanding of fraction addition and subtraction. The study found that over 60% of participants incorrectly assumed that adding fractions was the same as adding whole numbers, indicating a failure to transition from whole number reasoning to rational number concepts. These findings indicate a gap between the mathematical understanding expected at the university level and what students have acquired during their school years. While school mathematics often emphasizes procedural fluency—such as performing operations and solving standard problems - university mathematics demands a deeper conceptual understanding, problem-solving flexibility, and the ability to apply mathematical reasoning in unfamiliar contexts.

First-year university students are expected to transition from rote procedures to a more analytical and abstract approach to mathematics. They must be able to reason mathematically, interpret problems within real-world contexts, and engage in metacognitive reflection on their problem-solving strategies. However, when foundational misconceptions persist, such as misunderstandings in fraction equivalence, integer operations, or algebraic manipulation, students may struggle not only with introductory courses but also with the cumulative learning required in higher-level mathematics.



Therefore, identifying and understanding these misconceptions at the beginning of university study is a critical step toward designing effective instructional interventions. By addressing foundational errors early, educators can better support student learning trajectories and enhance academic success in mathematics and related disciplines. So, this study investigates the misconceptions held by first-year university students regarding basic mathematical concepts, with a focus on integer operations, fractions, and elementary algebra.

2. METHODS

This study employed a descriptive qualitative research design to explore misconceptions in basic mathematical concepts among first-year university students. The research was conducted at a private university located in Aceh, Indonesia. The participants consisted of 26 first-year students enrolled in the Mathematics Education program. These students were selected using purposive sampling, focusing on those who had recently completed senior secondary school and were enrolled in the Elementary Algebra course at the university level.

Data were collected through one main instrument, a diagnostic test. A written diagnostic test was developed consisting of 5 open-ended questions covering basic mathematical topics including operations with integers, fractions, and simple algebraic expressions. The questions were designed to reveal students' conceptual understanding and common errors.

The students' responses were analyzed using content analysis, focusing on identifying patterns of errors and classifying types of misconceptions. Misconceptions were categorized based on frameworks from prior studies, such as procedural misconceptions, conceptual misunderstandings, and representational difficulties. The interview data were transcribed and coded thematically to provide deeper insights into students' thought processes.

3. RESULTS & DISCUSSION

The analysis of the diagnostic test revealed that a significant number of students exhibited misconceptions related to basic mathematical operations. The most frequent misconceptions identified included:

3.1 Misunderstanding of Integer Operations

About 82% of students incorrectly answered questions involving negative integers and multiplication integers. Many students applied rules for addition, subtraction, and multiplication of integers inconsistently.

Figure 1 shows two examples of student work on integer operations. The left example shows a student calculating $27 \times (-13) = -381$, then subtracting 207 from -381 to get 174, and finally adding 115 to get 289. The right example shows a student calculating $27 \times (-13) = -351$, then subtracting 207 from -351 to get -558, and finally adding 115 to get -443.

Figure 1. Students' misunderstanding of integer operations

Based on Figure 1, the student demonstrates multiple conceptual errors, including: incorrect integer multiplication (-381 instead of -351), misinterpretation of negative subtraction (interpreting $-381 - 207$ as a positive value), and lack of consistency in sign rules, indicating possible reliance on intuition rather than rules. This example reflects a common misconception in integer operations, especially when dealing with negative numbers. Students often revert to treating operations as additive inverses without applying the correct rules for signed numbers. Instructional emphasis should be placed on: (a) clear understanding of negative multiplication



rules, (b) conceptual clarity about subtracting negative integers, and (c) consistent application of order of operations.

Misconceptions related to operations with integers, particularly involving negative numbers, often stem from early learning experiences that emphasize rules over understanding. Kwakye (2024) found that many pre-service mathematics teachers struggle with integer operations due to a weak conceptual grasp of negative values and their relationships. Students often memorize procedures (e.g., “two negatives make a positive”) without understanding why these rules work. Rosyidah et al. (2021) reported similar findings, showing that students frequently reverse signs or misapply rules when solving problems involving addition or subtraction of negative integers. These patterns suggest a reliance on rote procedures rather than conceptual reasoning.

3.2 Misconceptions in Fraction Operations

Approximately 90% of participants made errors in fraction addition and subtraction. A common error was directly adding numerators and denominators. This suggests a procedural rather than conceptual understanding of fractions.

Figure 2 consists of two photographs of student work. The left photograph shows a student's calculation: $2 \frac{3}{7} - 2 \frac{2}{5} + 3 \frac{2}{5}$. The student has written $= 27 - 14 + 16$, then $= 13 + 16$, and finally $= 29$. The right photograph shows a student's calculation: $2 \frac{3}{7} - 2 \frac{2}{5} + 3 \frac{2}{5}$. The student has written $= 12 - 9 + 10$, then $= 3 + 10$, and finally $= 13$. Both calculations show a clear misunderstanding of mixed numbers, where the whole number and numerator are being treated as separate whole numbers to be added or subtracted.

Figure 2. Students' misunderstanding of fraction operations

Based Figure 2. identified misconceptions and errors are:

(1) Misinterpretation of Mixed Numbers:

- Student incorrectly rewrote $2 \frac{3}{7}$ as 27 or 12, $2 \frac{2}{5}$ as 14 or 9, and so on.
- This suggests that the student may have concatenated the whole number and numerator, e.g., interpreting $2 \frac{3}{7}$ as 2 and 3 becoming 23 or 27, ignoring fractional meaning.

(2) Loss of Fractional Value:

None of the actual fractions (i.e., $\frac{3}{7}$, $\frac{2}{5}$) appear in the calculations, indicating a complete disregard for the denominator.

(3) Whole Number Bias:

This work illustrates a classic case of whole number bias, where the student treats all numbers as whole and simplifies the task to basic arithmetic with made-up whole numbers.

This student's answer demonstrates a severe misconception about mixed numbers and fractional reasoning. Instead of interpreting the expression as a combination of rational numbers, they mistakenly converted mixed numbers into unrelated whole numbers. This indicates a foundational gap in understanding the structure and value of fractions and mixed numbers, which needs to be addressed through concrete models and conceptual reinforcement.

Fraction operations are widely misunderstood, with many students erroneously adding numerators and denominators directly. This “whole number bias” has been well-documented. Jarrah et al. (2022) found that over 60% of middle school students in the UAE applied whole number strategies to fraction operations. Bentley and



Bossé (2018) similarly observed that college students retained these elementary misconceptions. According to Ni and Zhou (2005), this bias arises from treating fractions as two separate whole numbers rather than as a single rational quantity. As a result, students failed to recognize the need for common denominators or the logic behind fraction equivalence.

3.3 Errors in Applying Algebraic Rules

Around 76% of students misapplied algebraic rules as shown in Figure 3:

$$\textcircled{3} 13 - 8p = -27$$
$$-8p + 13 = 21 + 27$$
$$21 = 48$$

Figure 3. Students' errors in applying algebraic rules

Based on Figure 3, students' errors are as follows:

(1) Unclear Transformation:

- The expression $13 - 8p$ was rewritten as $-8p + 13$, which is algebraically acceptable due to the commutative property.
- However, the right-hand side was suddenly changed from -27 to $21 + 27$, which is not based on any valid algebraic step.

(2) Introduction of Arbitrary Numbers:

- The number "21" appears without explanation. This suggests the student may be guessing or attempting to force the equation into a simpler form.
- This represents a common error where students introduce numbers or steps that are not logically derived from the original equation.

(3) No Attempt to Solve for p :

- The student did not isolate p or continue solving the equation.
- The final answer "48" is not the value of the variable but seems to be the result of an unrelated computation.

(4) Misunderstanding of Equation Balancing:

- The student shows a lack of understanding of maintaining balance between both sides of an equation.
- This indicates a procedural misconception: knowing what steps to take, but not why or when to apply them.

The student demonstrated a fundamental misunderstanding of algebraic manipulation, particularly in solving linear equations. These mistakes are often due to an overreliance on memorized rules without meaningful connection to the logic of algebraic operations. Research also shows that students often misinterpret algebraic expressions and fail to apply rules consistently due to weak foundational knowledge (Kieran, 2007). The attempt



to rewrite and simplify lacks mathematical justification and reflects instrumental rather than relational understanding (Skemp, 1976). Instructional emphasis should focus on helping students understand the logic and structure of equation solving, ensuring that every step maintains equivalence and purpose.

These findings are consistent with previous research (e.g., Permata et al., 2019; Lee & Boyadzhiev, 2020), which also emphasized that students often bring misconceptions from earlier schooling that persist into higher education. Interviews confirmed that many students relied heavily on rote memorization without truly understanding the underlying concepts. For instance, one student explained that they “just followed the formula” when adding fractions but could not justify why the denominators must be the same.

Such misconceptions can hinder students’ ability to grasp more abstract algebraic concepts taught in university-level mathematics. Therefore, the results highlight the importance of early diagnostic assessment and conceptual reinforcement, particularly in courses like Elementary Algebra that bridge school and university mathematics.

4. CONCLUSION

This study revealed that first-year university students enrolled in the Elementary Algebra course at a private university in Aceh exhibited various misconceptions in basic mathematical concepts, particularly in integer operations, fractions, and simple algebraic expressions. These misconceptions, which likely originated during prior schooling, persist into higher education and may hinder students' ability to grasp more complex mathematical ideas.

The findings emphasize the need for early identification of conceptual misunderstandings through diagnostic assessments and for instructional strategies that prioritize conceptual understanding over procedural memorization. Strengthening students' foundational knowledge in mathematics is essential to support their academic success in university-level mathematics courses. Future studies are encouraged to explore intervention models or remedial programs that effectively address these persistent misconceptions and enhance students' mathematical reasoning skills.

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