EXPLORING MULTI-REPRESENTATION LEARNING APPROACH IN MATHEMATICS ELEMENTARY SCHOOL STUDENTS

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ABSTRACT

The purpose of this study was to analyze the effectiveness of the multi-representation learning approach in improving the understanding of mathematical concepts for elementary school students. The inclusion criteria in this review include studies that clearly apply a multi-representation approach in primary school settings and have relevant data to evaluate. Studies that do not provide adequate data or do not meet the inclusion criteria are excluded. Sources of information used to identify these studies include various academic databases such as ERIC, Springer, Elsevier, Ebschoohost, Taylor &; Francis, the ACM Technical Symposium on Computer Science Education and MDPI publishers. The last search was conducted last month before the writing of this abstract. Methods of evaluating bias risk in included studies include methodological quality assessment, data quality control, and evaluation of the accuracy of the statistical analysis used. The main findings of the study are presented through thematic and narrative analysis, with a total of 75 scientific studies included in this review. The characteristics of relevant studies will be discussed further in the analysis which includes research content, study design, type of participants, learning domain, and year of publication. The findings show that with various forms of representation students can improve their understanding of mathematics from informal to formal levels, in addition to the fact that multi-representation learning can deepen students' understanding of complex mathematical concepts.

Keywords: *Multi Representation Learning, Mathematics Elementary School, Algebraic Thinking*

1. INTRODUCTION

The use of various representations in mathematics education has been a major focus for many educators and is supported by NCTM Standards (NCTM, 1989). Mathematical ideas can be communicated and demonstrated in many different ways using representations (Greeno &; Hall, 1997; Pape & Tchoshanov, 2001). Mathematical representations, such as diagrams, graphs, tables, and symbols, are important for broadening students' understanding and improving their ability to solve mathematical problems. According to NCTM (1998), mathematics instructional programs should emphasize the use of representations to develop students' overall understanding of mathematics, enabling them to create, use, and communicate with mathematical representations effectively. Based on observational studies conducted and literature reviews, research studies on the use of multi-representation are implemented in mathematics learning, especially in teaching complex concepts such as algebra. Although various representations have been recognized as powerful tools for expanding students' understanding, challenges arise in determining how many representations should be used and how to properly integrate them in teaching. This is compounded by the complexity of the interaction between external and internal representations in the minds of students, which can affect the effectiveness of multi-representational learning.

Some research suggests that different combinations of external representations do not always produce the desired results and may even worsen students' understanding. Therefore, it is important to understand how to manage the complex interaction between external and internal representations as well as how to choose representations that best fit the learning context. This study aims to explore the effectiveness of multi-representation learning approaches in improving students' understanding of mathematical concepts, particularly in the context of algebraic thinking.

The results of research by Flores, R., Inan, F. A., Han, S., and Koontz, E. (2019) show advantages in performance from both teaching (MR and TA), and the use of multi-repressions has a statistically higher influence on students' understanding of traditional algorithms. In addition, the results of other studies by (Adadan, E., & Ataman, M. M. 2021) show that carefully designed inquiry instruction with multiple representations (IIMR) is

very useful for fostering students' scientific understanding of the particulate nature of matter (PNM) compared to regular instruction (RI). Instructors can use visual representations (1) to familiarize students with visual conventions commonly used in the math community, (2) to illustrate abstract complex concepts, (3) to enlarge the set of tools students have for engaging mathematics, and (4) to enhance students' subjective preferences and experience (see Acevedo Nistal et al., 2009; Singer 2007). Representations in the context of mathematics education may vary. Most researchers distinguish between external representations, such as graphs and tables, and internal representations, which are cognitive models that a person has (Janvier et al., 1993).

The multi-representation approach attracts attention because of the principle of multiple embodiments, which suggests that mathematical concepts must be presented in multiple forms in order for students to gain a deeper understanding. Although many studies have highlighted the importance of various representations in mathematics learning, challenges arise when deciding how many representations to use. Some research suggests that using too many representations can confuse students and hinder learning (Sweller et al., 1998). In addition, complex interactions occur between external representations and mental representations constructed by learners, affecting the effectiveness of multi-representational learning materials (Schnotz, 2014). In the context of more complex mathematical concepts, such as functions, learning with multiple representations plays a key role. External representations such as equations, graphs, tables, verbal descriptions, or real-world situations allow access to abstract concepts of functions (Duval, 2006). However, it is important to understand that the use of representations should be judicious, taking into account the complex interactions between external and internal representations as well as the needs of students to avoid confusing information overload. In this context, our research aims to explore the effectiveness of multi-representation learning approaches in enhancing students' understanding of mathematical concepts. Through a systematic literature review, we will identify trends, findings, and pedagogical implications for enriching mathematics learning practices at the primary school level

2. METHODS

The research focused on analyzing various articles that have been published in scientific journals in the field of mathematics education. The approach used is systematic literature review (SLR) by following the steps outlined in the Preferred Reporting Items for Systematic Review and Meta-Analysis (PRISMA) flow chart (Page et al., 2018). The process within the PRISMA framework includes the stages of identification, screening and inclusion as illustrated in Diagram 2. In the initial stage (identification), a search was carried out on 45 articles found through various academic databases such as ERIC, Springer, Elsevier, Ebschoohost, Taylor & Francis, the ACM Technical Symposium on Computer Science Education, and MDPI publishers. The search was conducted using the keywords "multi representation learning approach" and "Multiple Representation" for the period 2010 to 2023. Then, a selection was made based on the title of the article and abstracts that were relevant to the context of mathematics education. In the next stage (screening), articles are classified based on criteria such as content suitability, study design, type of publication, learning domain, and year of publication. The third stage (included), then the selected articles are 10 and then compiled for further investigation in the literature review.
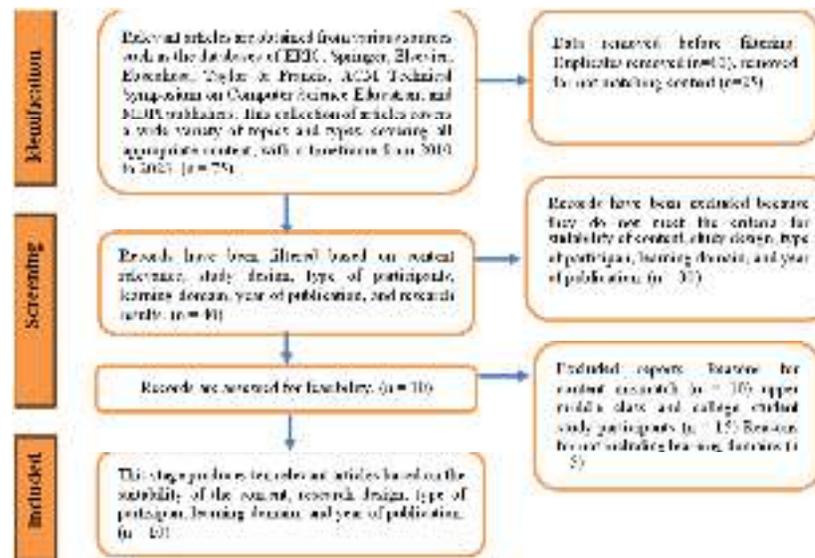
Figure 1. PRISMA Statement
(Page et al., 2018)

2.1   Data Sources

The data sources in this study are ERIC, Springer, Elsevier, Ebscohost, Taylor &; Francis databases, the ACM Technical Symposium on Computer Science Education, and MDPI Publisher. The following explanation is contained on the pages of each database: 1) ERIC is a comprehensive, easy-to-use, searchable, Internet-based bibliographic and full-text database of education research and information. It is sponsored by the Institute of Education Sciences within the U.S. Department of Education. https://eric.ed.gov/?faq; 2) Springer We are a world-leading research, educational, and professional publisher. Visit our main website for more information. https://www.springernature.com/gp/authors; 3) Elsevier As a global leader in information and analytics, we help researchers and healthcare professionals advance science and improve health outcomes, striving to create a better future worldwide. https://www.elsevier.com/about; 4) EBSCOhost is an intuitive online research platform used by thousands of institutions and millions of users worldwide. With quality databases and search features, EBSCOhost helps researchers of all kinds find the information they need fast. https://www.ebsco.com/products/ebscohost-research-platform; 5) Taylor & Francis is one of the world's leading academic publishers and one of its most enduring. https://taylorandfrancis.com/about/; 6) The Technical Symposium addresses problems common among educators working to develop, implement and/or evaluate computing programs, curricula, and courses. The symposium provides a forum for sharing new ideas for syllabi, laboratories, and other elements of teaching and pedagogy, at all levels of instruction. https://sigcse.org/events/symposia/index.htm; 7) MDPI is a member of the Committee on Publication Ethics (COPE). MDPI takes on the responsibility of enforcing rigorous peer review together with strict ethical policies and standards to ensure the addition of high-quality scientific works to the field of scholarly publication. https://www.mdpi.com/about

2.2   Research Instrument

Research Instruments The instrument used for this study is to classify related aspects observed in Table 1. There are five main aspects of this study. Research content, study design, type of participants, learning domain, and year of publication. Furthermore, researchers make exceptions to the aspects of data collection instruments and analytical methods provided, since this aspect has been very common in previous studies. These existing categories are discussed in Table 1, and then the research design is divided into two categories, namely quantitative and qualitative, and not applied to the general research conducted. The subjects of the study include,

Elementary School (SD), and junior high school (SMP). The treatment provided includes a multi-representation approach.

Table 1. Research Instrument

Aspects	Category
Content Research	1. Characterizing the growth of one student's mathematical understanding in a multi-representational learning environment. 2. Using the onto-semiotic approach to analyze novice algebra learners' meaning-making processes with different representations. 3. Interacting with indeterminate quantities through arithmetic word problems: Tasks to promote algebraic thinking at elementary school 4. Analysis of peer learning behaviors using multiple representations in virtual reality and their impacts on geometry problem solving 5. Mono-and multi-representational learning of the covariational aspect of functional thinking. 6. Abstraction through multiple representations in an integrated computational thinking environment. 7. Improvement Algebraic Thinking Ability Using Multiple Representation Strategy on Realistic Mathematics Education 8. Three Phase Ranking Framework for Assessing Conceptual Understanding in Algebra Using Multiple Representations. 9. Visual representations in mathematical word problem solving among form four students in Malacca 10. On the promises and pitfalls of collaborative learning with multiple representations.
Study Design	1. Experiment 2. Experiment 3. Experiment 4. Experiment 5. Experiment 6. Experiment 7. Quasi Experiment 8. Qualitative Analysis 9. Qualitative Analysis 10. Qualitative Analysis
Type of Participants	1. Junior High School 2. Junior High School 3. Junior High School 4. Junior High School 5. Junior High School 6. Junior High School 7. Junior High School 8. Primary School 9. Primary School 10. Primary School
Learning Domain	1. Algebraic Thinking 2. Algebraic Thinking 3. Algebraic Thinking 4. Algebraic Thinking 5. Computational Thinking 6. Functional Thinking

Aspects	Category
	7. Geometry Problem Solving 8. Mathematics Understanding 9. Problem Solving 10. Problem Solving
Year of Publication	1. 2010 2. 2010 3. 2011 4. 2013 5. 2018 6. 2020 7. 2020 8. 2022 9. 2022 10. 2023

2.3　Data Analysis

Data Analysis Each article that has been identified has been classified according to carefully established criteria, paying attention to every predetermined aspect. It is important to note that each article must meet every predefined category to get the classification. This decision-making process is based on a thorough evaluation of the information available in the abstract, research methods, discussion, and research results of each article. In the next stage, the data that has been collected and classified is presented in the form of a bar chart. This approach aims to ensure that data is presented systematically, transparently, and documented in detail, thus enabling comprehensive understanding and accurate analysis.

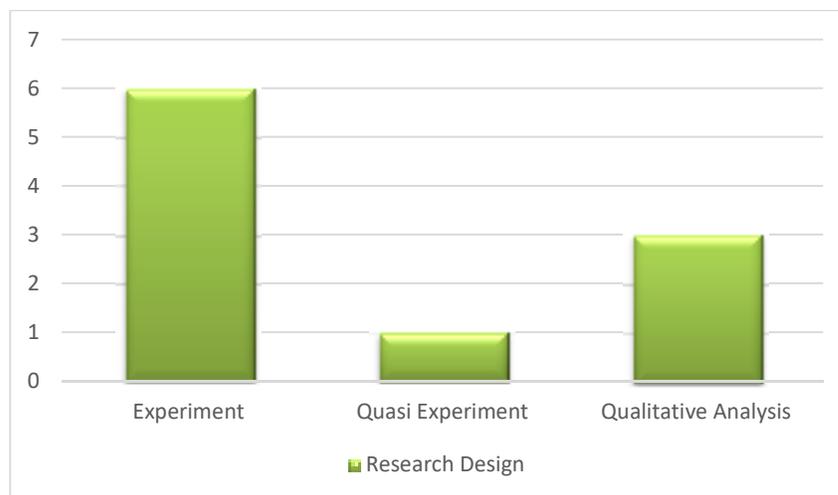## 3.　RESULTS & DISCUSSION

3.1　Research Design

Figure 2. Research Design

Based on Figure 3.1 from several studies that have been analyzed, in general, as many as six articles using experimental design, the focus is on observing the effects or impact of a multi-representation approach on students' thinking skills in a mathematical context. This involves measuring the effectiveness of multi-

representation methods in helping primary school students understand mathematical concepts. Quasi-experimental articles, however, do not fully control for the factors affecting the dependent variable. Researchers are concerned about the impact of multi-representation learning approaches on students' math achievement. Furthermore, through research using qualitative analysis methods, researchers can explore more in-depth aspects of student and teacher experiences in adopting a multi-representation learning approach. Qualitative analysis allows the disclosure of complex patterns in perceptions, attitudes, and learning experiences, which cannot always be measured directly through experimental approaches.

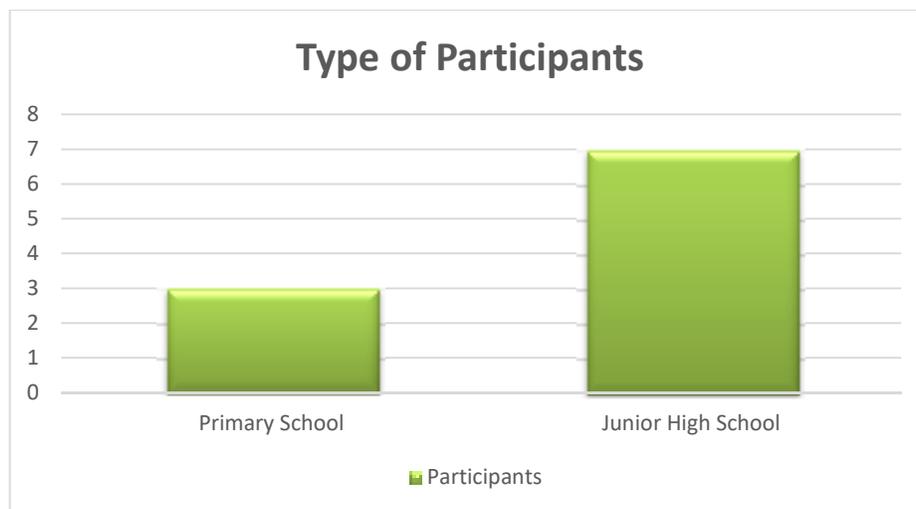### 3.2   *Type of Participant*

Figure 3. Type of Participants

Based on Figure 3, 2 types Participants in high school who dominate more than elementary school students may have different cognitive development characteristics, such as shorter concentration skills, limited understanding of abstractions, and varying levels of motivation. Therefore, in interpreting findings from articles involving primary school students, it is important to take into account how a multi-representation approach can be adapted to the level of cognitive development and individual needs of this population. On the other hand, junior high school participants may have more mature cognitive abilities, but may also face new challenges in understanding more abstract mathematical concepts. Thus, an analysis of articles focusing on junior high school students should consider how a multi-representation approach can provide appropriate support in overcoming these barriers and deepening the understanding of complex mathematical concepts. In addition, differences in learning environments, teaching methodologies, and curriculum approaches among elementary and junior high school students can also influence how they respond to multi-representation learning approaches.

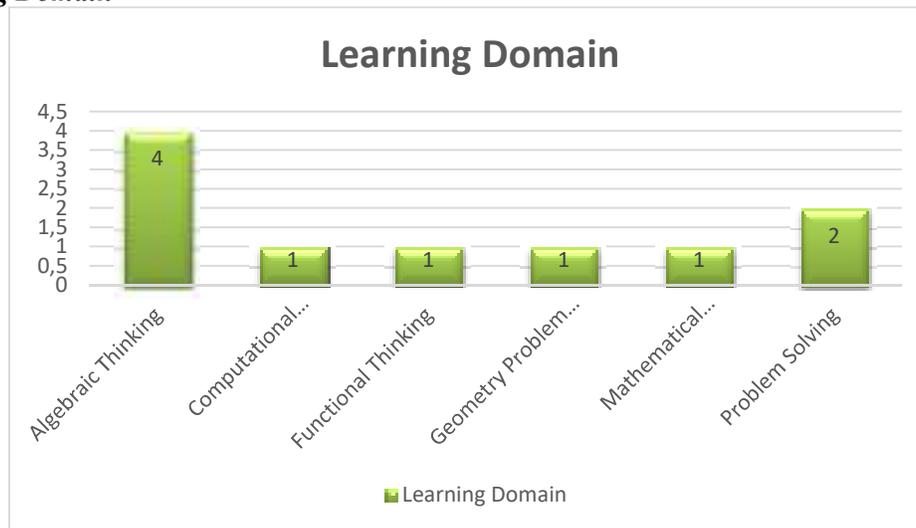### 3.3   *Learning Domain*

Figure 4. Learning Domain

Based on Figure 3.3, the learning domain refers to the diverse aspects of mathematics that are the focus of the multi-representation learning approach explored. Given the variation in learning domains among the articles investigated, complex interpretations are needed to understand how multi-representation approaches can be effectively applied in primary school math contexts. Articles highlighting learning domains such as algebraic thinking explore students' understanding of fundamental algebraic concepts, such as patterns, relationships, and generalizations. Interpretation of findings from these articles requires an in-depth analysis of how visual, symbolic, and verbal representations can be used to facilitate a better understanding of algebraic concepts for elementary school students. Meanwhile, articles on computational thinking and functional thinking bring an additional dimension to the understanding of mathematical concepts, with an emphasis on problem solving and the use of mathematics in a broader context. Complex interpretations of these articles may involve analysis of how a multi-representation approach can enrich students' computational and functional skills, as well as how this contributes to the development of more in-depth mathematical problem-solving. Furthermore, articles highlighting geometry problem solving explore students' ability to apply geometry concepts to solve problems relevant to the real world. The interpretation of this article can pay attention to how visual representations of geometric objects and spatial relationships can improve students' understanding and ability to solve geometry problems. Finally, articles that discuss problem solving in general provide insight into the use of a multi-representation approach in facilitating the solving of mathematical problems in general, without restrictions on specific learning domains. Complex interpretations of these articles may require analysis of how multi-representation approaches can be adapted to meet problem-solving needs in a variety of mathematical contexts.
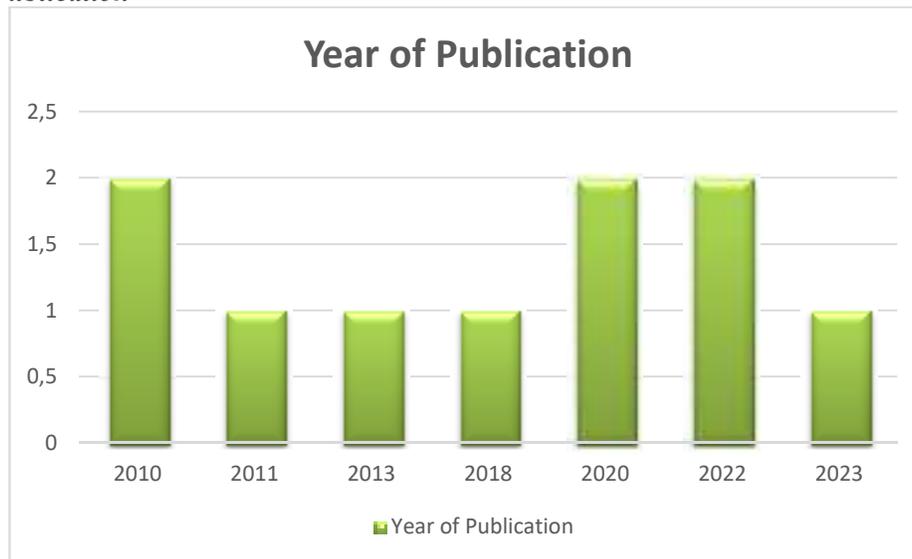
3.4 Year of Publication

Figure 5. Year of Publication

Figure 3.4 in this study reflects the evolution and development of multi-representation learning approaches in the context of mathematics education for elementary school students. Taking into account variations in the distribution of articles by year of publication, we can see how this research has evolved over time and how the findings can be understood in the context of the development of mathematics learning theory and practice. Articles published in 2010 reflect early attempts to understand the potential and challenges of this approach, as well as gain early insight into its impact on students' mathematical understanding. Later, articles published in 2011 and 2013 continued to explore the multi-representation learning approach, with further research on its application and effectiveness in various contexts of mathematics learning. The development of methodologies and theories underlying the multi-representation learning approach, as well as efforts to overcome obstacles that arise in its implementation. In 2018, the single article may reflect the growing interest in multi-representation learning approaches, with researchers focusing on specific aspects such as problem-solving development or technology integration in mathematics learning. Interpretations of this article may take into account growing trends in the literature and attempt to answer more specific research questions in the domain of multi-representation learning. Finally, the articles published in 2020 reflect the peak of research interest and activity in this field, with researchers expanding the scope of their research to cover a wide range of aspects of a multi-representation learning approach by highlighting the most relevant and cutting-edge trends and findings in the literature.

3.5 Research Results

The results showed that students were able to advance in their mathematical understanding from the informal level to the formal level in the Pirie-Kieren model. This shows that the multi-representation learning approach has a positive impact on the development of students' mathematical understanding (Gulkilik, H., Moyer-Packenham, P. S., Ugurlu, H. H., & Yuruk, N., 2020). Other findings highlight that verbal representations have a significant influence on students' abilities, and their interaction with other representations can affect students' mathematical comprehension (Erbilgin, E., & Gningue, S. M., 2023). Reflection on the interpretation of equations has also been shown to favor the identification of meanings associated with indeterminate quantities, contributing to students' understanding of mathematical concepts (Ayala-Altamirano, C., Pinto, E., Molina, M., & Cañadas, M. C., 2022). Multi-representation learning, such as virtual manipulatives, written mathematical formulas, and verbal explanations, is proven to help students build mathematical concepts and develop critical thinking (Hwang, W.

Y., & Hu, S. S., 2013). The influence of learning with representation is evident as a process of complex interaction between learning content and forms of representation, highlighting the importance of the integration of representation in the learning process (Rolfes, T., Roth, J., & Schnotz, W., 2022). Multi-representation in the curriculum can transform a student's ability from learning abstractly to learning through abstraction, showing a positive impact on student understanding (Gautam, A., Bortz, W., & Tatar, D., 2020). Students who use multiple representation strategies are shown to have better algebraic thinking skills, emphasizing the importance of using multi-representation in math learning (Kusumaningsih, W., & Herman, T., 2018). Multi-representation has been shown to improve students' understanding of linear equations, demonstrating the relevance and effectiveness of this learning approach in the context of high school mathematics (Panasuk, R. M., 2010). Nonetheless, less than two percent of problems were solved using pictorial representations, suggesting that students' preferences tended toward more schematic solutions (Ahmad, A., Tarmizi, R. A., & Nawawi, M., 2010). Managing challenges in the use of mathematical representations requires regular opportunities for students to reconsider their roles and use flexible tools in solving complex mathematical tasks (White, T., & Pea, R., 2011).

The results showed that students were able to improve their mathematical understanding from informal to formal levels in the Pirie-Kieren model. This suggests that a multi-representation learning approach has a positive impact on the development of students' mathematical understanding. Other findings suggest that verbal representations have a significant influence on students' ability to understand math, with their interactions influencing students' understanding of the concepts. Reflection on the interpretation of equations has also proven useful in helping students identify the meanings associated with complex mathematical concepts. Multi-representation learning, which includes the use of virtual manipulatives, written mathematical formulas, and verbal explanations, has proven effective in helping students build mathematical concepts and improve their critical thinking skills. The influence of learning with representation suggests that it is a complex process of interaction between learning content and forms of representation, highlighting the importance of the integration of representation in mathematics learning. The application of multi-representation in the curriculum has a positive impact on students' understanding, transforming their ability from learning abstractly to learning through abstraction. Students who use multiple representation strategies are shown to have better algebraic thinking skills, emphasizing the importance of using multi-representation in math learning. Multi-representation has also been shown to improve students' understanding of linear equations, demonstrating the relevance and effectiveness of this learning approach at the secondary school level. Nonetheless, students' preferences tend towards more schematic solutions rather than using pictorial representations. Overcoming challenges in using mathematical representations requires regular opportunities for students to reconsider their roles and use flexible tools in solving complex mathematical tasks.

4. CONCLUSION

In this study, an analysis of the literature has been carried out that discusses the exploration of multi-representation learning approaches among elementary school students in mathematics subjects. Over the last thirteen-year period, there has been a significant increase in the number of publications adopting a multi-representation learning approach. The articles found tend to be dominated by experimental studies. The main research subjects were junior high school students, with special emphasis on elementary school students. The most prominent finding is related to the development of algebraic thinking skills in students. The findings suggest that with different forms of representation, students can improve their understanding of mathematics from an informal level to a formal level. In addition, the results of multi-representation learning can deepen students' understanding of complex mathematical concepts.

5. ACKNOWLEDGMENTS

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